

Gravitino dark matter in gauge mediated supersymmetry breaking

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This paper investigates the parameter space of theories with gauge mediated supersymmetry breaking leading to gravitino (cold) dark matter with mass $m_{3/2} \sim 1 \text{ keV} \rightarrow 10 \text{ MeV}$. We pay particular attention to the cosmological rôle of messenger fields. Cosmology requires that these messengers decay to the visible sector if the lightest messenger mass $M_X \gtrsim 30 \text{ TeV}$. We then examine the various possible messenger number violating interactions allowed by the symmetries of the theory and by phenomenology. Late messenger decay generally results in entropy production hence in the dilution of pre-existing gravitinos. We find that in $SU(5)$ grand unification only specific messenger-matter couplings allow to produce the required amount of gravitino dark matter particles. Gravitino dark matter with the correct abundance is however expected in larger gauge groups such as $SO(10)$ for generic non-renormalizable messenger-matter interactions and for arbitrarily high post-inflationary reheating temperatures.

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I. INTRODUCTION

Supersymmetric extensions of the standard electroweak model of particle physics come with a variety of appealing byproducts, such as the stabilization of the Higgs mass against radiative corrections, radiatively induced electro-weak symmetry breaking at the electro-weak scale, and the possibility of $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge coupling unification at a sufficiently high energy scale. Most of these features occur rather naturally after supersymmetry (SUSY) has been broken softly, and it is believed that ultimately such a breaking has to occur spontaneously within some theory describing all four fundamental interactions.

Despite the lack of a particularly compelling model, there is a number of proposals with various theoretical and phenomenological merits, where the spontaneous SUSY breaking usually takes place dynamically in a (hidden) sector of the theory which does not contain the standard model particles. Models can be classified according to the origin of SUSY breaking and of the soft terms, i.e. how the breaking of supersymmetry is transmitted to the low energy (visible) sector. In the so-called gauge mediated supersymmetry breaking (GMSB) models, this transmission is induced by renormalizable gauge interactions [1, 2] (see [3] for a review). Particularly attractive features of these scenarios are the natural suppression of neutral current flavor changing interactions as well as a highly predictive mass spectrum that will be put to test in forthcoming collider experiments.

In theories with gauge mediation, the lightest supersymmetric particle (LSP) is the gravitino and its mass can lie anywhere in the range $m_{3/2} \sim 1 \text{ eV} \rightarrow 1 \text{ GeV}$. Such a light gravitino (i.e. when $m_{3/2} \gtrsim 1 \text{ keV}$) is traditionally associated with a cosmological catastrophe. Indeed many studies have examined the production of light gravitinos in the early Universe in order to place stringent bounds on the post-inflationary reheating temperature (see [4, 5, 6] and references therein). Few studies have contemplated the possibility that this gravitino LSP could make up the dark matter of the Universe [7, 8, 9, 10, 11, 12], mainly because in the most naive model one needs to adjust the reheating temperature as a function of the gravitino mass in order to obtain the required dark matter relic density. However Refs. [7, 8] have recognized the important cosmological rôle of the messenger particles that are part of the spectrum of all GMSB theories. In particular, it has been shown that the late decay of the lightest messenger to visible sector particles can induce a substantial amount of entropy production which would result in the dilution of the predicted gravitino abundance. As a result, the light gravitino problem could be turned into a light gravitino blessing, i.e. one would obtain suitable gravitino dark matter for arbitrarily high reheating temperatures.

These studies [7, 8] have focused on two specific couplings between the messenger and visible sectors. This motivates us to examine in more generality the possibility of producing the right amount of gravitino dark matter in GMSB scenarios. We do so in the present paper by con-

sidering all messenger-matter interactions allowed by the gauge symmetries of the theory and by phenomenology and by considering their impact on the gravitino abundance. The present study thus aims at being more exhaustive than prior investigations; on the way we will also improve on some results previously obtained. In particular we show that the coupling introduced in Ref. [7] does not appear in minimal GMSB models and that multi-goldstino production channels modify substantially the results of Ref. [8]. Finally we also take into account stringent constraints from big-bang nucleosynthesis and large scale structure formation.

Our study is similar in spirit to those conducted for neutralino dark matter in minimal supergravity [13]. However since the present paper is of an exploratory nature, we approximate the mass spectrum of GMSB models by three parameters: the messenger mass scale M_X , the supersymmetric particles mass scale $M_{\text{SUSY}} \sim 1$ TeV and the gravitino mass $m_{3/2}$. In particular, we treat $m_{3/2}$ and M_X as the fundamental parameters in our search for gravitino dark matter. We also discuss the influence of the nature and mass of the next-to-lightest supersymmetric particle (NLSP). Furthermore we calculate the velocity of the dark matter gravitinos in order to examine whether this dark matter is hot, warm or cold. For simplicity we assume that R -parity holds. However, we will show that, when $m_{3/2} \lesssim 10$ MeV, our results remain valid even if R -parity is violated. In this range the gravitino lifetime becomes much longer than the age of the Universe, so that it can be considered as stable on our cosmological time and constraints from diffuse background distortions are eluded [14, 15]. Finally we note that Ref. [16, 17] has discussed the possibility of gravitino dark matter in minimal supergravity models (with gravity mediation of supersymmetry breaking). The cosmology of these models is different from that of GMSB scenarios as the gravitino is heavier ($m_{3/2} \gtrsim 10$ GeV) and there are no messenger particles. It is found in these studies that only a limited region of parameter space can satisfy the big-bang nucleosynthesis constraints and that the reheating temperature must be tuned in order to obtain the required dark matter relic density.

The plan of this paper is as follows. In Section II we review the basics of gauge mediation models and discuss the nature of the lightest messenger particle which plays a crucial rôle in our analysis. In Section III we discuss the numerous channels of light gravitino production in the early Universe as well as gravitino dilution due to late decay of a massive particle and the relevant cosmological constraints. In Section IV we survey the various renormalizable and non-renormalizable messenger number violating interactions allowed by the gauge symmetry of the theory and discuss their consequences with respect to the light gravitino problem and gravitino dark matter. Finally in Section V, we discuss various perspectives, notably with respect to the case of $SO(10)$ grand unification. We restrict ourselves to GMSB scenarios in the framework of $N = 1$ $D = 4$ supergravity and use natural

units $\hbar = c = k_B = 1$; $m_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV denotes the reduced Planck mass.

II. MESSENGER SECTOR

A. Gauge mediation of SUSY breaking

Gauge-mediated SUSY breaking (GMSB) [2, 3], is usually implemented by adding a term

$$W = S\Phi_M\bar{\Phi}_M + \Delta W(S, Z_i) \quad (1)$$

to the superpotential where Φ_M and $\bar{\Phi}_M$ are messenger left chiral superfields with $SU(3) \times SU(2) \times U(1)$ quantum numbers, whereas the spurion left chiral superfield S and the secluded sector Z_i fields are electroweak and strong- interactions singlets. Upon the development of a non-vanishing vev $\langle S \rangle$ of the scalar component of the spurion superfield and a SUSY-breaking expectation value of the spurion auxiliary field F_S , due to unspecified dynamics in the secluded sector $\Delta W(S, Z)$, fermionic messengers combine into Dirac fermions of mass $M_{X,1/2} = M_X \equiv \langle S \rangle$, whereas their bosonic partners mix in a mass matrix of eigenstates ϕ and $\bar{\phi}$ with masses $M_\phi = M_X(1 - F_S/M_X^2)^{1/2}$ and $M_{\bar{\phi}} = M_X(1 + F_S/M_X^2)^{1/2}$. In terms of the messengers bosonic components, $\phi = (-\Phi_M^* + \bar{\Phi}_M)/\sqrt{2}$ and $\bar{\phi} = (\Phi_M + \bar{\Phi}_M^*)/\sqrt{2}$. Note that ϕ denotes a set of scalar fields transforming under some representation of the grand unified gauge group, and that the mass degeneracy of this multiplet is lifted by D -terms and radiative corrections.

Since the messengers share the standard model gauge interactions, the gaugino and scalar spartners acquire mass at the one- and two-loop levels respectively:

$$\tilde{m}_{1/2} \sim \left(\frac{\alpha}{4\pi}\right) \frac{F_S}{M_X}, \quad \tilde{m}_0^2 \sim \left(\frac{\alpha}{4\pi}\right)^2 \left(\frac{F_S}{M_X}\right)^2, \quad (2)$$

hence the quantity $\Lambda = F_S/M_X$ is the supersymmetry breaking scale in the visible sector. Provided $F_S/M_X \approx 100$ TeV, this generates the required order of magnitude for the soft parameters¹. Note that $F_S < M_X^2$ is mandatory otherwise one of the messengers bosons acquires a negative mass squared. This also implies $M_X \gtrsim 100$ TeV, and for $M_X \gg 100$ TeV, $F_S \ll M_X^2$, hence M_X sets the mass scale for the messenger sector. In particular, $M_\phi \approx M_X$, and in the following no distinction will be made between these two mass scales, except where otherwise noted.

The gravitino mass is related to the fundamental SUSY breaking scale, $m_{3/2} \equiv F/\sqrt{3}m_{\text{Pl}}$, with $F \equiv F_S + \sum_i F_{Z_i}$

¹ In the present exploratory study, we do not address the detailed features of the GMSB mass spectrum and the related electroweak symmetry breaking and fine-tuning issues.

the sum of F -terms in the secluded sector. We define the parameter $k \equiv F_S/F \leq 1$, so that $m_{3/2} = F_S/(k\sqrt{3}m_{\text{Pl}})$. In direct gauge mediated scenarios, one expects $k \lesssim 1$, whereas in scenarios in which the transmission of supersymmetry breaking to the messenger sector is loop suppressed one may find $k \ll 1$. Note that one can also relate the parameters k , M_X and $m_{3/2}$ via the following formula: $m_{3/2} = \Lambda M_X/(k\sqrt{3}m_{\text{Pl}})$. Since Λ is tied to the electroweak scale, the latter equation allows to eliminate one parameter, which we choose to be k , in terms of $m_{3/2}$ and M_X , which we will treat as the fundamental parameters.

B. Lightest messenger

Taken at face value, GMSB scenarios generically lead to a cosmological catastrophe, as they predict that the lightest messenger should overclose the Universe². In effect, messenger gauge interactions as well as those derived from Eq. (1) conserve messenger number so that the lightest messenger (a boson) is stable in this minimal version of the theory. As messengers can be easily produced in the primordial plasma thanks to their gauge interactions, their present day abundance is given by the result of a thermal freeze-out of messenger annihilation (akin to the well-known neutralino LSP freeze-out in gravity mediated SUSY breaking).

Through explicit computation, one can show that the lightest messenger generically overcloses the Universe unless its mass $M_X \lesssim 10^4 \text{ GeV}$ [18]. By lightest messenger, it is understood the lightest component of ϕ after taking into account D -terms and radiative corrections. Henceforth we denote this component by X . It has been shown that if the messengers sit in $\mathbf{5} + \bar{\mathbf{5}}$ representations³ of $SU(5)$, the lightest messenger carries the gauge charges of a sneutrino $\tilde{\nu}_L$, and its relic abundance would be of the right order of magnitude provided its mass $\sim 10 - 30 \text{ TeV}$ [18, 19]. Note that the mass scale in the messenger sector is *a priori* unconstrained, since phenomenology constrains the ratio F_S/M_X as discussed previously. If the messengers sit in $\mathbf{10} + \bar{\mathbf{10}}$ representations of $SU(5)$, the lightest messenger carries the gauge charges of a selectron \tilde{e}_R . Charged dark mat-

ter is forbidden by cosmology [20] and moreover this messenger would overclose the Universe for typical values of M_X . Finally, if the messengers sit in $\mathbf{16} + \bar{\mathbf{16}}$ representations of $SO(10)$, the lightest messenger is a $\tilde{\nu}_R$ -like $SU(3) \times SU(2) \times U(1)$ singlet. The next-to-lightest messenger can only decay to the lightest messenger by GUT scale suppressed interactions, and its lifetime of order $10^{10} \text{ yrs } (M_X/100 \text{ TeV})^{-5}$ is so long that its decay produces unacceptable distortions of the diffuse backgrounds [18].

Cosmology thus forbids the lightest messenger to be stable unless messengers can be diluted to a very low abundance or the lightest messenger happens to have mass $M_X \sim 10 - 30 \text{ TeV}$. If the post-inflationary reheating temperature is larger than M_X and no late-time entropy production occurs, then messenger number must be violated, i.e. the Lagrangian of the theory must contain additional messenger-matter interactions. However such terms can spoil the phenomenological successes of the minimal model, in particular the absence of flavor changing neutral currents or an adequate pattern of electroweak symmetry breaking. One is thus tempted to believe that such further messenger interactions with visible sector particles are rather weak, possibly resulting from non-renormalizable operators. This will be discussed in more detail below.

Delayed messenger decay can have dramatic consequences for the gravitino problem and/or the possibility of gravitino dark matter. If a non-relativistic species comes to dominate the energy density of the early Universe and subsequently decays into visible sector particles, a secondary epoch of reheating results and is concomitant with the dilution of any pre-existing relics, such as gravitinos. The abundance of these relics may then, even for “arbitrarily” high primary reheat temperatures of the Universe after an inflationary epoch, be in accord with current observational constraints [7, 8, 9].

A crucial element in this analysis is the messenger abundance before decay. This is given, as mentioned earlier, by the thermal freeze-out of messengers, hence by their annihilation cross-section. The mass splitting in the messenger multiplet is generally small, of order $F_S/M_X \ll M_X$, hence one should in principle consider the various co-annihilation channels. This task is however left to a future more refined study; we note that over most of parameter space the inclusion of co-annihilation channels should modify the relic abundance of the lightest messenger by at most a factor of order unity since the various particles that would co-annihilate have comparable annihilation cross-sections [18].

1. Annihilation cross-section

Annihilation through gauge interactions. In the case of $SU(5)$ unification, Dimopoulos *et al.* [18] have calculated the annihilation cross-section of the lightest messenger through gauge interactions for $\mathbf{5} + \bar{\mathbf{5}}$ representations

² Obviously, similar problems can in principle arise also for the Z_i fields present in ΔW if the lightest Z_i mass is larger than $\sim 100 \text{ TeV}$ [18]. However, the issue becomes much more model-dependent here, and we will thus assume for simplicity that secluded sector fields can decay rapidly to the spurion field S . The latter field is free from such cosmological problems: even though it can be either heavier or lighter than the messengers [see the discussion following Eq.(3)], in the first case it decays at tree-level to messengers, while in the second it decays to gauge bosons and gauginos fairly quickly through one-loop effects which are not suppressed by supersymmetry.

³ Messengers sitting in complete GUT representations preserve automatically gauge coupling unification.

and parametrized it as:

$$\langle \sigma_{XX^*} v \rangle \simeq \frac{1}{M_X^2} \left(A + \frac{B}{x} \right), \quad (3)$$

with $x \equiv M_X/T$ and $A \simeq -B \simeq 3 \cdot 10^{-3}$. This calculation takes into account all annihilation channels (mediated by gauge interactions) to gauge bosons, Higgses, neutralinos, charginos, fermions and sfermions, see Ref. [18].

Annihilation into two goldstinos. The above calculation neglects annihilation into two goldstinos $XX^* \rightarrow \tilde{G}\tilde{G}$. This latter occurs through a variety of diagrams: in the t - and u - channels, the annihilation takes place through the exchange of the fermionic mass eigenstate partner of X . In the s - channel, the annihilation occurs through the exchange of a graviton, a spurion and other secluded sector scalar particles. Finally, annihilation also occurs through four-point contact interactions $XX^*\tilde{G}\tilde{G}$. These various contributions are triggered by various operators in the supergravity Lagrangian, taking into account the goldstino component of the gravitino Ψ_μ and the fermionic spurion ψ_S fields after supersymmetry breaking as follows:

$$\Psi_\mu = i\sqrt{\frac{2}{3}} \frac{\partial_\mu \tilde{G}}{m_{3/2}} + \dots \quad (4)$$

$$\psi_S = \frac{F_S}{F} \tilde{G} + \dots \quad (5)$$

For instance, the four-point contact interactions $XX^*\tilde{G}\tilde{G}$ derive from the gravitino mass term in the supergravity Lagrangian after expanding the exponential of the Kähler function to second order in the scalar around their vev. The Yukawa coupling between the goldstino and the hidden sector SUSY breaking scalars Z_i , which enters the s -channel exchange diagrams, also derives from this expansion. The coupling XX^*Z_i derives from the scalar potential trilinear couplings. Finally, the coupling between X , its fermions mass eigenstate partner and \tilde{G} is obtained directly from the supergravity Lagrangian coupling between the gravitino and a pair of fermion-boson partners, taking the appropriate linear combination to express it in terms of the mass eigenstates after SUSY breaking.

The annihilation cross-section into two goldstinos must be calculated with care since some leading high energy contributions are expected to cancel out [21, 22, 23]. It will be important to distinguish between the cases where the spurion mass M_S is larger or smaller than that of the lightest messenger. Both configurations are dynamically possible: for instance, one finds in the simplest models [2] that $M_X^2 = (\kappa - \sqrt{3}\lambda)(\kappa/\lambda^2)M_S^2$, where λ and κ denote respectively the spurion self-coupling and its coupling to the messenger fields in the superpotential, and where we have neglected here the effect of the spurion coupling to the secluded sector fields following the study of [24] for the stability and local minima conditions. In this case

the spurion is heavier than the lightest messenger when $\sqrt{3} \leq \kappa/\lambda \lesssim 2.2$ and lighter when $\kappa/\lambda \gtrsim 2.2$.

In the light scalars and non-relativistic limit, $s \sim 4M_X^2(1 + 3/x) \gg m_{Z_i}^2$ with m_{Z_i} mass of the secluded sector scalar Z_i , and $x = M_X/T$ as above, the cross-section reads:

$$\langle \sigma_{XX^* \rightarrow \tilde{G}\tilde{G}} v \rangle = \left(\frac{F_S}{F} \right)^4 \frac{1}{32\pi M_X^2} \left(1 - \frac{15}{4x} \right). \quad (6)$$

In effect, performing an expansion in F_S/s in the matrix element, one finds that both terms of order 0 and 1 cancel among the various contributions, yielding a cross-section $\propto F_S^4$. Its high energy limit $s \gg M_X^2, m_{Z_i}^2$ is actually that of spurion-spurion annihilation into two goldstinos [23]. In practice, Eq. (6) applies if the spurion is much lighter than the lightest messenger since its coupling to XX^* dominates that of secluded sector scalars as a result from the tree-level coupling in the superpotential between S , Φ_M and $\bar{\Phi}_M$. However, in this limit one must also account for annihilation of X, X^* into spurions; this will be discussed further below.

If secluded sector scalars are heavier than the lightest messenger, the limit $s \ll m_{Z_i}^2$ applies and in this case, one can neglect the s -channel exchange of these scalars. The cancellation between the various diagrams occurs only to order 0 in (F_S/s) , leaving a cross-section $\propto F_S^2$:

$$\langle \sigma_{XX^* \rightarrow \tilde{G}\tilde{G}} v \rangle \simeq \frac{1}{8\pi} \frac{F_S^2 M_X^2}{F^4} \left(1 - \frac{3}{2x} \right). \quad (7)$$

The s -channel exchange graphs of the secluded sector scalars are suppressed by $s/m_{Z_i}^2$. Note that the term F_S in these expressions should be understood as the mass squared difference between the fermion and boson components of the lightest messenger multiplet, rather than as the *vev* of the auxiliary component of S . These two quantities differ if the superpotential includes a coupling constant κ , $W \supset \kappa S \Phi \bar{\Phi}$; our choice here is $\kappa = 1$.

The annihilation cross-section in the heavy spurion limit [Eq. (7)] violates the unitarity bound $\langle \sigma_{\text{ann}} v \rangle \lesssim 8\pi/M_X^2$ [25] (in the non-relativistic regime) for $M_X \gtrsim 1.63 \cdot 10^7 \text{ GeV} (m_{3/2}/1 \text{ keV})^{2/3}$. Beyond this limit the effective Lagrangian is no longer valid, and one expects sizeable contributions from multi-goldstinos production. Hence the results obtained hereafter in the region where the unitarity bound is violated are highly uncertain and model-dependent. In what follows, we assume that the cross-section saturates at the unitarity limit in this region $M_X \gtrsim 1.6 \cdot 10^{11} \text{ GeV} (m_{3/2}/1 \text{ GeV})^{2/3}$ if the spurion is heavier than the lightest messenger. We also consider the other possible limit in which the cross-section follows Eq. (6), so that the comparison of these two cases will allow us to assess the impact of the above effects on the relic gravitino abundance.

Since phenomenology requires that $M_X \sim F_S/10^5 \text{ GeV}$ (see Section II.A), it is easy to see that annihilation into goldstinos dominates the cross-section in the heavy scalars limit for $M_X \gtrsim 3 \cdot 10^6 \text{ GeV} (m_{3/2}/1 \text{ keV})^{2/3}$.

Hence the inclusion of this channel in the present calculation modifies rather drastically the relic abundance of the lightest messenger in this part of parameter space.

Annihilation into two spurions. If S is lighter than X , there is no problem associated with unitarity, and one can safely use Eq. (6) all throughout parameter space. One must nonetheless account for $XX^* \rightarrow SS$ annihilation, whose cross-section reads

$$\langle \sigma_{XX^* \rightarrow SS} v \rangle = \frac{1}{64\pi M_X^2} \left(1 - \frac{1}{x} + \left(-\frac{3}{2} + \frac{23}{4x} \right) r \right) \quad (8)$$

to first order in x^{-1} and in $r (\equiv M_S^2/M_X^2)$, and where we neglected for simplicity the contribution of a λS^3 term in the superpotential, assuming that $\lambda \ll \kappa \simeq 1$.

This cross-section is comparable to the annihilation cross-section through gauge interactions given in Eq. (3) and results in the decrease of the relic abundance of the lightest messenger by a factor of order 2. For direct GMSB models in which $F_S \sim F$, the annihilation channel into goldstinos becomes dominant and must be taken into account.

2. Relic abundance

Freeze-out of the lightest messenger annihilations occurs at a value x_f :

$$x_f \simeq \log \left[Q_f \left(1 + \frac{B/A}{\log(Q_f)} \right) \frac{1}{\sqrt{\log(Q_f)}} \right], \quad (9)$$

where $Q_f \simeq 6.1 \cdot 10^{10} (M_X/10^6 \text{ GeV})^{-1} A$, and the values of A and B accounts for the various possible channels depicted above. In terms of this freeze-out value x_f , the relic abundance is then given by:

$$Y_X \simeq 2.1 \cdot 10^{-14} \left(\frac{M_X}{10^6 \text{ GeV}} \right) \frac{x_f}{A + B/2x_f}. \quad (10)$$

In the case of messenger sitting in $\mathbf{10} + \overline{\mathbf{10}}$ representations of $SU(5)$, the relic abundance is expected to be similar to the above to within a factor of a few, since the lightest messenger carries hypercharge. For simplicity, we thus assume that its relic abundance is also given by Eq. (10) above.

Finally, in the case of $SO(10)$ grand unification, the lightest messenger is a singlet under the standard model gauge interactions. As argued in Ref. [26], it can annihilate through one-loop diagrams (which dominate the exchange of tree level GUT mass bosons considered in Ref. [18]) and into two goldstinos at tree level as above. This case has been discussed in some detail in the low M_X region in Ref. [26]. In Section V we sketch briefly the parameter space of $SO(10)$ GMSB scenarios using order of magnitude estimates of these diagrams. In the main discussion of this paper, we thus focus on $SU(5)$ grand

unification with messengers either in $\mathbf{5} + \overline{\mathbf{5}}$ or $\mathbf{10} + \overline{\mathbf{10}}$ representations.

The cosmological scenario we have in mind is the following. As the Universe reheats to high temperature after inflation, radiation along with gravitinos and messengers are produced. As the temperature decreases, the lightest messenger annihilations cease and its abundance freezes-out. This non-relativistic lightest messenger may come to dominate the energy density if its decay to the visible sector is sufficiently delayed and its relic abundance sufficiently large. In particular, messengers come to dominate the energy density when the background temperature

$$T_{\text{dom}} \simeq \frac{4}{3} M_X Y_X \quad (11)$$

(provided $T_{\text{dom}} \gtrsim T_{\text{dec}}$, with T_{dec} the temperature at which messengers decay, see below). During decay of the lightest messenger, gravitinos and possibly sparticles may be produced, but the pre-existing gravitino and NLSP abundances are diluted by entropy production. Finally, at late cosmological times, the NLSP decays to the gravitino. In some cases the NLSP may decay before the lightest messenger. The final gravitino abundance is then the sum of gravitinos produced by sparticle and messenger interactions at early times and diluted later by the appropriate factor due to messenger decay, plus gravitinos produced during the secondary reheating induced by messenger decay as well as gravitinos produced in the decay of the NLSP. Then, for a given messenger decay width, which characterizes the dilution factor, one may find in the $m_{3/2} - M_X$ plane the region where satisfactory abundances for gravitinos are found.

If $T_{\text{dom}} < T_{\text{dec}}$, the lightest messenger never comes to dominate the energy density and no entropy production ensues. This is what has been generally assumed in previous studies that derived upper limits on the post-inflationary reheating temperature from the upper limit $\Omega_{3/2} < 1$, e.g. [4, 6]. This case is discussed in Section IV.A.1.

III. GRAVITINO PRODUCTION

A. Production channels

The fractional contribution to the present critical density of non-relativistic gravitinos \tilde{G} with number-to-entropy ratio $Y_{3/2} = n/s$ is given by

$$\Omega_{3/2} h^2 = 2.81 \cdot 10^8 \left(\frac{m_{3/2}}{1 \text{ GeV}} \right) Y_{3/2} \quad (12)$$

where h is the Hubble constant in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The number-to-entropy ratio $Y_{3/2}$ is found by following the Boltzmann equation describing

gravitino production:

$$\frac{dY}{dT} = \frac{1}{sHT} \left(\sum_i \langle \Gamma_{i \rightarrow \tilde{G} + \dots} n_i \rangle + \sum_{i,j} \langle \sigma_{i+j \rightarrow \tilde{G} + \dots} v_{ij} n_i n_j \rangle \right), \quad (13)$$

which includes production by sparticle decays $\Gamma_{i \rightarrow \tilde{G} + \dots}$ and scatterings $\langle \sigma_{i+j \rightarrow \tilde{G} + \dots} v_{ij} n_i n_j \rangle$, neglecting three-body and higher order interactions. In principle, one should take into account gravitino annihilation as well. However it is sufficient to approximate possible gravitino losses by imposing that the gravitino number-to-entropy ratio never exceeds its thermal equilibrium value $Y_{\text{eq}} \simeq 3.7 \times 10^{-3} (g_*/230)^{-1}$, with g_* the number of relativistic degrees of freedom in the thermal bath⁴.

The production of gravitinos in the early Universe is dominated by the production of the helicity $\pm 1/2$ component if $m_{3/2} \ll M$, where M denotes the mass scale of particles leading to gravitino production. This helicity $\pm 1/2$ component is related to the goldstino through Eq. (4), associated with the breaking of local SUSY, where Ψ_μ and \tilde{G} denote respectively the helicity $\pm 1/2$ gravitino components and the goldstino [27]. In scenarios of gauge mediated SUSY breaking in which $m_{3/2} \lesssim 1 \text{ GeV} \ll M_{\text{SUSY}} \sim 1 \text{ TeV}$, the correspondence $\Psi_\mu \sim i\sqrt{2/3} \partial_\mu \tilde{G}/m_{3/2}$, is generically satisfied. The production of goldstinos may then be decomposed into the contributions from particle scatterings, decay of particles before the freeze-out of the NLSP, and the possible contribution of NLSP decay.

The decay and scattering contributions of visible sector fields have been calculated in various studies using the above gravitino-goldstino equivalence (see e.g., Ref. [5] for a detailed discussion and references). Later on it has been shown that the contribution of messengers is quite significant [6]. Indeed messengers couple directly to the goldstino \tilde{G} through the superpotential term $S\Phi_M\bar{\Phi}_M$ and the fraction of goldstino \tilde{G} comprised in the fermionic component of S , Eq. (5). We use the decay widths and cross-sections for messengers interactions leading to gravitino production given in Ref. [6].

These authors have also argued that the goldstino decouples from the visible sector fields at energies $M_X \lesssim E \lesssim \sqrt{F}$ claiming that the effective gravitino-particle-sparticle vertex is induced by loop diagrams involving messengers [28]. However this analysis has been performed in the limit of global supersymmetry and it thus ignores the tree level fermion-sfermion-goldstino $-(1/\sqrt{2}m_{\text{Pl}})g_{ij*}\partial_\mu\Phi^{*j}\bar{\chi}_R\gamma^\nu\gamma^\mu\Psi_\nu + \text{h.c.}$ and gauge boson-gaugino-goldstino $(i/4m_{\text{Pl}})\bar{\Psi}_\mu\sigma^{\rho\sigma}\gamma^\mu\lambda_L^{(a)}[F_{\rho\sigma}^{(a)} + \tilde{F}_{\rho\sigma}^{(a)}]$ [29] interaction terms which appear in local supersymmetry. Therefore the goldstino does not decouple from visible

sector fields at energies $M_X \lesssim E \lesssim \sqrt{F}$, at variance with Ref. [6]. Nevertheless, it is true, following Refs. [6, 28] that production channels should include the loop-induced messenger contribution to the effective particle-sparticle-goldstino vertex at energies $E \lesssim M_X$. This is done in the present study; we do not include thermal corrections to the cross-sections since they are found to be negligible [30].

The various contributions to $Y_{3/2}$ have different temperature dependences. The largest fraction of the decay contributions results from cosmic epochs when the temperature falls below the mass of the corresponding particles. In contrast the dimension-5 operators associated to visible sector particle – sparticle scatterings are most effective at high temperatures; the contribution from messengers peaks at temperatures $T \sim M_X$. A simple fit to the results of Ref. [6], with the modifications according to the remarks above, gives an estimate of the amount of gravitinos produced by scatterings and decays of sparticles and messengers for a reheating temperature T_{RH} :

$$\begin{aligned} \Omega_{3/2} h_{70}^2 \sim & 2 \cdot 10^{-4} \left(\frac{m_{3/2}}{1 \text{ GeV}} \right)^{-1} \left[0.6 \left(\frac{M_3}{10^3 \text{ GeV}} \right)^3 \right. \\ & \left. + 2.4 \left(\frac{M_3}{10^3 \text{ GeV}} \right)^2 \left(\frac{T_{\text{RH}}}{10^5 \text{ GeV}} \right) \right] \\ & + 2 \left(\frac{m_{3/2}}{1 \text{ GeV}} \right)^{-1} \left[3 \left(\frac{M_3}{10^3 \text{ GeV}} \right)^4 \left(\frac{M_X}{10^5 \text{ GeV}} \right)^{-1} \right. \\ & \left. + 0.2 \left(\frac{M_3}{10^3 \text{ GeV}} \right)^2 \left(\frac{M_X}{10^5 \text{ GeV}} \right) \right]. \quad (14) \end{aligned}$$

The first two lines correspond to sparticle decays and particle-particle scatterings, respectively, while the last two correspond to interactions involving messenger fields. Hence the latter should only be included if $T_{\text{RH}} \gtrsim M_X$. In these equations, M_3 denotes the gluino mass scale, which controls the amount of gravitinos produced. Indeed, the coupling between gauginos, gauge bosons and goldstinos dominates the particle-sparticle-goldstino contributions at high temperatures and scales as the gauge coupling constant; its contribution is represented by the second term on the r.h.s. of Eq. (14). Decay contributions to $\Omega_{3/2} h^2$ scale as M^3 , with M the mass of the decaying sparticle, see the first term on the r.h.s. of Eq. (14). The gluino appears in this term due to the larger degree of freedom for colored particles than for color singlets as well as due to colored sparticles being heavier than color singlets in the ratio α_3/α_2 or α_3/α_1 [see Eq. (2)]. Strictly speaking this relation applies at the messenger scale and must be corrected by renormalization group running at lower energy scales. Nevertheless the mass spectrum of GMSB models is dominated by squarks and gluinos whose masses are comparable at scale M_X . As regards the contribution of messengers, given in Eq. (14) for $T_{\text{RH}} \gtrsim M_X$, the gluino mass appears only in the combination $(4\pi/\alpha_3)M_3 \sim F_S/M_X$,

⁴ Note that this equilibrium abundance only includes spin 1/2 goldstinos, with the population of 3/2 components of the gravitino assumed to be negligible.

i.e. as the normalization of F_S as a function of M_X and the electroweak scale.

We provide Eq. (14) in order to assist the reader in interpreting the figures that follow. The results shown below are obtained from the integration of the Boltzmann equation. Also recall that Eq. (14) does not take into account all contributions to the gravitino abundance. One must notably add the contribution of gravitinos produced by annihilations of the lightest messenger, as well as lightest messenger decay and NLSP decay.

The NLSP decays into final states including one gravitino with width:

$$\Gamma_{\text{NLSP} \rightarrow \tilde{G}} \simeq \frac{1}{48\pi} \frac{M_{\text{NLSP}}^5}{m_{3/2}^2 m_{\text{Pl}}^2}. \quad (15)$$

The background temperature at NLSP decay is then $T_{\text{NLSP} \rightarrow \tilde{G}} \simeq 5 \text{ MeV} (M_{\text{NLSP}}/100 \text{ GeV})^{5/2} (m_{3/2}/1 \text{ MeV})^{-1}$. This decay occurs late and consequently big-bang nucleosynthesis constraints on hadronic or electromagnetic energy injection at time $\sim 10^{-2} - 10^8 \text{ sec}$ lead to the exclusion of a significant part of parameter space, as will be seen in the following. Since one NLSP produces one gravitino, the gravitino yield of NLSP decays in terms of entropy density is simply Y_{NLSP} .

In principle, the NLSPs result from a freeze-out of thermal equilibrium, hence Y_{NLSP} can be calculated in the same way as the relic abundance of the lightest messenger. However, one must not forget the possible production of NLSPs during the decay of the lightest messenger. If the decay temperature of the latter is larger than the NLSP freeze-out temperature, this contribution is washed out by NLSP annihilations. However if decay occurs later, NLSPs are regenerated to a level which depends on the time of messenger decay and on the annihilation cross-section of the lightest messenger. In more detail, if the lightest messenger decays to NLSPs before NLSPs decay in turn to gravitinos, the NLSPs have time to annihilate. In Ref. [8], a procedure was outlined to calculate the number density of NLSPs remaining after these further annihilations. If the lightest messenger decays to NLSPs after pre-existing NLSPs have decayed to gravitinos, the calculation of the final number of gravitinos produced is more involved and necessitates the integration of coupled Boltzmann equations. For simplicity, we assume that all NLSPs produced in messenger decay then decay instantaneously to gravitinos without annihilating. This maximizes the number of gravitinos produced hence reinforces the constraints from relic density arguments, and, in this sense, this assumption is conservative. Keeping track of NLSP annihilation before decay to gravitinos would not affect our results significantly as it is marginal in most of parameter space [8].

Finally there exist other potential channels of gravitino production. One is that of helicity $\pm 3/2$ production, which for large gravitino mass and high reheating temperature may become important. The helicity $\pm 3/2$ modes interact with gravitational strength only

and are produced by interactions in the thermal bath in abundance $\Omega_{3/2}^{\pm 3/2} h^2 \sim (m_{3/2}/1 \text{ GeV})(T_{\text{RH}}/10^{14} \text{ GeV})$ [5]. Therefore this contribution does not dominate in most of the $m_{3/2} - M_X - T_{\text{RH}}$ parameter space. We do not take into account possible non-thermal production channels of helicity $\pm 3/2$ gravitinos during inflation [31]. The amount of gravitinos produced in this way depends strongly on the underlying model of inflation, even though it may exceed the amount of helicity $\pm 3/2$ modes produced by scatterings in the thermal bath in particular models.

B. Gravitino Dilution

The delayed decay of non-relativistic messengers may have dramatic consequences on the abundance of any pre-existing species, such as gravitinos. In case delayed decay results in the temporary matter-domination by messenger rest mass, i.e. $\rho_X \gg \rho_r$ where ρ denote energy densities and subscripts X and “r” refer to messenger and radiation, respectively, entropy production is significant and results in the severe dilution of any pre-existing number-to-entropy ratio Y . In this case the post-messenger-decay cosmic radiation temperature $T_{\text{dec}}^>$ is substantially larger than the pre-decay temperature $T_{\text{dec}}^<$, akin of a second reheat. Approximating decay to be instantaneous when the Hubble scale equals the decay width of the lightest messenger, $H \approx \Gamma_X$, one finds

$$T_{\text{dec}}^> \approx (g_> \pi^2/90)^{-1/4} \sqrt{\Gamma_X m_{\text{Pl}}}, \quad (16)$$

where $g_>$ denotes the number of relativistic d.o.f. at temperature $T_{\text{dec}}^>$. If the particle decays into the visible and into an invisible sector, the decay width in Eq. (16) above should be multiplied by B_{visible} , with B_{visible} the branching ratio into visible sector particles. This may be of relevance notably when the lightest messenger decays into visible sector particles and into gravitinos which do not share their energy density with the visible sector afterwards.

By equating the pre- and post-decay energy densities, the pre-decay radiation temperature $T_{\text{dec}}^<$ is obtained in terms of $T_{\text{dec}}^>$ and T_{dom} [see Eq.(11)] at which X comes to dominate the energy density, as:

$$T_{\text{dec}}^< \approx T_{\text{dec}}^> \min \left[1, \left(\frac{g_>}{g_<} \right)^{1/3} \left(\frac{T_{\text{dec}}^>}{T_{\text{dom}}} \right)^{1/3} \right]. \quad (17)$$

Obviously, if X does not dominate the energy density before decaying, $T_{\text{dec}}^< \approx T_{\text{dec}}^>$, while if $T_{\text{dec}}^> \ll T_{\text{dom}}$, one finds $T_{\text{dec}}^< \ll T_{\text{dec}}^>$ and entropy production is very substantial. In effect the ratio of pre-decay and post-decay entropy densities, gives the entropy release $\Delta_X \equiv s_>/s_< = g_> T_{\text{dec}}^3 / g_< T_{\text{dec}}^3$:

$$\Delta_X \approx \max \left[1, \frac{T_{\text{dom}}}{T_{\text{dec}}^>} \right], \quad (18)$$

The values of $T_{\text{dec}}^>$ and T_{dom} are given in terms of Y_X , M_X and Γ_X through Eqs. (16) and (11).

Such entropy release dilutes pre-existing densities according to: $Y_> = Y_</math>. Nevertheless, it should be borne in mind that, in case of high second reheat temperatures $T_{\text{dec}}^>$, the regeneration of diluted species may occur. This effect is taken into account in our calculations by treating messenger decay as a second reheat. Note that substantial entropy release after BBN is unacceptable, and $T_{\text{dec}}^> \gtrsim 1 \text{ MeV}$ is required. Eq. (16) may thus be employed to infer a fairly strict lower limit of $\Gamma_X \gtrsim 4.3 \times 10^{-25} \text{ GeV}$ on abundant and slowly decaying particle species in the early Universe.$

Particularly interesting to cosmology is the case of significant entropy dilution. In this limit one may use Eqs. (18) and (17) to derive the entropy dilution factor for an arbitrary species with mass M_X , decay width Γ_X and abundance Y_X , in the limit $\Delta_X \gg 1$:

$$\begin{aligned} \Delta_X &\approx 0.77 g_>^{1/4} Y_X \Gamma_X^{-1/2} m_{\text{pl}}^{-1/2} M_X \\ &\approx 28 \left(\frac{M_X}{10^8 \text{ GeV}} \right) \left(\frac{Y_X}{10^{-10}} \right) \left(\frac{\Gamma_X}{10^{-25} \text{ GeV}} \right)^{-1/2} \left(\frac{g_>}{10} \right)^{1/4} \end{aligned} \quad (19)$$

where it is understood that if $\Delta_X \lesssim 1$ is found, it ought to be substituted by $\Delta_X = 1$.

If there exists a whole tower of N unstable, but long-lived particles, with abundances Y_i , masses M_i and decay widths Γ_i for particle i , the final dilution factor is determined solely by the properties of the slowest decaying messenger. In particular, all the equations above may be employed as if any prior decays had not occurred.

C. Cosmological constraints

1. Hot, warm or cold dark matter

Collisionless damping during the radiation era leads to the erasure of power in density fluctuations below a length scale (the free-streaming scale) that is mostly determined by the time at which dark matter particles become non-relativistic, or equivalently by their velocity extrapolated to zero redshift. A particle of mass m that thermally decouples from the plasma when relativistic has a present-day velocity:

$$\langle v_0 \rangle \approx 0.018 \text{ km s}^{-1} (g_{\star, \text{dec}}/230)^{-1/3} (m/1 \text{ keV})^{-1}, \quad (20)$$

with $g_{\star, \text{dec}}$ the number of d.o.f. at decoupling. Cosmological data on the power spectrum of density fluctuations allow to place constraints on the mass of the particle.

One finds $m \gtrsim 1 \text{ keV}$ from the requirement that the Universe has reionized by $z \sim 6$ [32] or from the measurement of the power spectrum in the Lyman α forest [33]. If reionization has occurred as early as $z \sim 17$, as suggested by the recent WMAP data [34], then a mass larger than $\sim 10 \text{ keV}$ seems required [35].

These constraints are important to our analysis and we keep track of the average velocity extrapolated to $z = 0$. Obviously the limits between hot, warm and cold matter are fuzzy, and we choose to qualify as warm dark matter particles with velocity $0.0018 \text{ km s}^{-1} \leq v_0 \leq 0.054 \text{ km s}^{-1}$, corresponding to particle masses $0.3 \text{ keV} \leq m \leq 10 \text{ keV}$ (for freeze-out from thermal equilibrium as above). For velocities above the upper limit or below the lower limit, we mean hot or cold dark matter respectively. It is important to note that entropy production after decoupling of the gravitinos cools down these dark matter particles according to: $\langle v_0 \rangle \rightarrow \langle v_0 \rangle / \Delta_X^{1/3}$.

For gravitinos produced by out-of-equilibrium processes, notably by the late decay of a massive particle, the above relation between mass and velocity is modified. Assuming the outgoing gravitino carries a momentum of half the mass M of the decaying particle, the present velocity reads:

$$\langle v_0 \rangle \approx (M/2m_{3/2})(3.91/g_{\star, \text{dec}})^{1/3} (T_0/T_{\text{dec}}), \quad (21)$$

with $g_{\star, \text{dec}}$ the number of d.o.f. at decay, T_0 the present cosmic background temperature and T_{dec} the temperature at decay. Note that T_{dec} generally depends on M so that the dependence between the nature of gravitino dark matter (cold/warm/hot) and the mass of the decaying particle is not necessarily trivial. For instance, one can show that a decaying NLSP produces hot/warm dark matter if its mass $\lesssim 500 \text{ GeV}$.

If the decay occurs at temperatures sufficiently high that the gravitino can interact and thermalize, one should rather use Eq. (20). However at temperatures $T \ll 100 \text{ GeV}$ the gravitino has decoupled from the thermal plasma [22], mainly because the sparticles have decoupled themselves. Therefore the decay of the NLSP or of the lightest messenger (if sufficiently late) generally produces highly relativistic gravitinos.

2. Big-bang nucleosynthesis constraints

Due to the different channels of gravitino production, one generally finds that gravitinos are made of two generic sub-populations: one that has been produced by equilibrium processes and another made of hot gravitinos produced by out-of-equilibrium decays. It may be that the latter are so highly relativistic that they form a hot dark matter component. However their impact on the formation of large scale structure may be negligible if their contribution to the gravitino energy density is negligible. In this particular case, constraints from big-bang nucleosynthesis on extra degrees of freedom may apply and constrain this population. We take

the BBN constraints on extra degrees of freedom to be $\delta g \leq 1.8$ (corresponding to 1 extra neutrino family allowed) [36]. Gravitinos that are relativistic at the time of BBN and carry energy density $\rho_{3/2}^s$ contribute to the level $\delta g_{\text{BBN}}/g_{\text{BBN}}^s \simeq \rho_{3/2}^s/\rho_r^s$, with ρ_r^s the standard radiation energy density at the onset of BBN with $g_{\text{BBN}}^s = 10.75$. Gravitinos that were once in thermal equilibrium or that were produced by equilibrium processes at temperature T (d.o.f. g) carry characteristic momentum $p_{3/2} \sim 3T_{\text{BBN}}(g_{\text{BBN}}^s/g^s)^{1/3}$ at BBN, with $T_{\text{BBN}} \sim 1$ MeV. Assuming $g^s \sim 230$, these gravitinos are relativistic if $m_{3/2} \lesssim 1$ MeV and their contribution to the energy density is $\delta g_{\text{BBN}}/g_{\text{BBN}}^s \simeq Y_{3/2}^r$ with $Y_{3/2}^r = n_{3/2}^r/s$, hence it is negligible due to the upper bound on $Y_{3/2}$ resulting from thermal equilibrium.

However most relativistic gravitinos at the time of BBN result from out-of-equilibrium decays, e.g. from NLSP or from the lightest messenger decay. Given that, immediately after decay the outgoing gravitinos carry a fraction $B_{3/2}$ of the rest mass energy of the decaying particle of mass M as kinetic energy, their contribution to the energy density at the onset of BBN reads: $\delta g_{\text{BBN}}/g_{\text{BBN}}^s \simeq B_{3/2}MY_M(4/3)(g_{\text{BBN}}^s/g^s)^{1/3}/T_d$, where T_d is the decay temperature and Y_M the number-to-entropy ratio of the parent at decay. Here it has been assumed that $T_d \gtrsim 4MY_M/3$. In the opposite limit, i.e. in the case of significant entropy production at decay, the above relation becomes $\delta g_{\text{BBN}}/g_{\text{BBN}}^s \simeq B_{3/2}(g_{\text{BBN}}^s/g^s)^{1/3}$ (assuming $B_{3/2} \ll 1$), and it will be this limit which results in the strongest BBN constraints.

The time of decay of NLSPs to gravitinos is also strongly constrained by big-bang nucleosynthesis limits on hadronic and electromagnetic energy injection at times $\sim 10^{-2} \rightarrow 10^8$ sec. These constraints have been examined in Refs. [17, 37, 38, 39], while Ref. [40] has translated these bounds on the messenger scale M_X of GMSB models assuming $k = 1$ (which is equivalent to setting an upper bound on $m_{3/2}$). In the present analysis, we use the latest constraints from hadronic and electromagnetic energy injection from Ref. [38].

In GMSB scenarios, the NLSP is generically a neutralino (mainly bino) or a stau. The former decays predominantly into a photon and a goldstino; the fraction of energy spent with branching ratio $B_{\text{em}} \simeq 1$ and by three body decays into a pair of quarks and goldstino with hadronic branching ratio $B_{\text{had}} \sim 10^{-3}$; if its decay to Z bosons is not suppressed by phase space, i.e. $(M_{\text{NLSP}} - M_Z)/M_Z \gtrsim 1$, the hadronic branching ratio $B_{\text{had}} \sim 0.15$. For simplicity, we use this latter value, which is conservative with respect to the constraints inferred. Concerning the annihilation cross-section of the bino NLSP, we use the value $\langle \sigma_{\text{NLSP}} v \rangle = 10^{-9} \text{ GeV}^{-2} (M_{\text{NLSP}}/100 \text{ GeV})^{-2}$, which corresponds to the bulk region of minimal supergravity [17]. We will comment on the dependence of our results on the choices made when discussing the results shown in Fig. 1 below.

A stau NLSP may produce in its decay electromagnetic and hadronic showers. About 100% of the energy

is converted to electromagnetically interacting particles. In 70% of all decays, a stau NLSP produces hadrons, but these are mesons whose lifetimes are so short that they do not have time to interact before decaying if they were emitted at times $\gtrsim 10^2$ sec. Hence, we use $B_{\text{had}} = 0.7$ for stau decay timescales shorter than 10^2 sec and $B_{\text{had}} = 10^{-3}$ for longer decay times. The stau annihilation cross-section is not as model dependent as that of the bino, $\langle \sigma_{\text{NLSP}} v \rangle \simeq 10^{-7} \text{ GeV}^{-2} (M_{\text{NLSP}}/100 \text{ GeV})^{-2}$. Given its large annihilation cross-section, the stau has a small relic abundance, and consequently the BBN constraints are comparatively weaker.

Overall big-bang nucleosynthesis constraints apply to a combination of Y_{NLSP} (relic abundance) and decay timescale $\propto M_{\text{NLSP}}^{-5} m_{3/2}^2$. Note that for a decay timescale $\tau \sim 10^3$ sec, interesting modifications to BBN may result [39]. We also note that in a very limited part of parameter space of GMSB theories, the NLSP can be a sneutrino [3], for which the BBN constraints would be largely reduced [17, 41].

Finally, since the mass of the NLSP enters the BBN constraints while the gravitino yield is controlled by the gluino mass, it is necessary to schematize the mass spectrum of GMSB scenarios. We do so by assuming a mass ratio $M_3/M_{\text{NLSP}} \sim 6$ [3, 42] and fiducial values $M_{\text{NLSP}} = 150 \text{ GeV}$ and $M_3 = 1 \text{ TeV}$. Where relevant we mention the possible influence of these values on our results.

IV. MESSENGER COUPLINGS TO MATTER AND GRAVITINO DARK MATTER

In this section, we investigate the possible solutions for gravitino dark matter for various messenger number violating interactions added to the Lagrangian. As argued in Section II, such interactions are mandatory if no substantial entropy production occurs at temperatures $\lesssim M_X$ (other than due to lightest messenger domination and decay) in order to avoid the cosmological problems that would result from the stability of the lightest messenger. For definiteness, we adopt the notations of Ref. [29] including four component spinors, in the general supergravity Lagrangian.

A. Renormalizable couplings

1. Superpotential couplings

Renormalizable couplings, beyond those of the required messenger gauge interactions, may exist, though they are constrained by considerations of flavor changing neutral currents as well as the potential development of charge- and color- breaking minima, among other issues. Dine *et al.* [43] have analysed viable extensions of the minimal GMSB scenario in this direction, introducing

couplings of the form

$$W \supset y_l^i H_D \Phi_M^l \bar{e}_i + y_q^i H_D Q_i \bar{\Phi}_M^q, \quad (22)$$

where Φ_M^l and $\bar{\Phi}_M^q$ denote lepton- and quark-like messengers. Here Φ_l denotes an SU(2) messenger doublet, the y_i 's are Yukawa couplings with family index i , and H_D , \bar{e} , and Q_i are standard model down-type Higgs, right-handed lepton, and quark doublet, respectively.

Additional SUSY-breaking mass splittings are generated by these types of interactions via one-loop contributions yielding, for example, negative relative mass contributions of order $\delta m_{\tilde{e}}/m_{\tilde{e}} \approx -10^3 y_l^2 F_S^2/M_X^4$ to slepton masses. Flavor changing neutral currents may place potentially restrictive limits on such couplings, as due to the experimentally verified weakness of such processes, mass splittings between 1st- and 2nd- generation sleptons (and squarks) are constrained to be smaller than $(m_{\tilde{e}_1}^2 - m_{\tilde{e}_2}^2)/m_{\tilde{e}}^2 \lesssim 10^{-3}$. Assuming conservatively, $y_l^1 \neq 0$ and $y_l^2 = y_l^3 = 0$ one may thus infer a limit $y_l^1 \lesssim (M_X/10^8 \text{ GeV})$ on this extra Yukawa coupling.

Interactions induced by the superpotential Eq. (22) also induce the decay of messengers, in particular $X \rightarrow \tilde{H}^- e^+$ assuming X carries the same gauge charges as a $\tilde{\nu}_L$ (see Section II). Hence the decay width $\Gamma_X = y^2 M_X/8\pi$. Though the limit on Yukawa couplings as inferred above may be quite severe, entropy production due to delayed messenger decay is absent when terms of Eq. (22) are included into W , unless the extra Yukawa coupling is extremely small, $y \lesssim 10^{-15} (M_X/10^7 \text{ GeV})^{3/2}$, assuming $SU(5)$ grand unification.

It is nevertheless instructive to study the influence of such “fast” decay of the lightest messenger on the possibility of having gravitino dark matter. As mentioned earlier, this case (without entropy production) has been implicitly assumed in previous studies that have drawn upper bounds on the post-inflationary reheating temperature from the upper bound on the gravitino density $\Omega_{3/2} < 1$ [4, 6] (and references therein). In order to provide a point of comparison with this previous literature, we plot in Fig. 1 the results of the calculation of $\Omega_{3/2}$ in the plane $T_{\text{RH}} - m_{3/2}$, using the techniques developed in the previous section. We assume “fast” decay with width $\Gamma_X \sim 10^{-9} M_X$ corresponding to $y \sim 10^{-4}$, which ensures that phenomenological constraints are satisfied for all values of M_X . The results shown are insensitive to the exact value of y , provided it is not so tiny that substantial entropy production would occur.

The shade (color) coding in this figure and all subsequent figures is as follows: lightest (yellow) corresponds to $\Omega_{3/2} < 0.01$ (no gravitino problem but no dark matter), and the increasingly darker (respectively green, red and blue) areas indicate respectively the regions of cold, warm and hot dark matter in which $0.01 < \Omega_{3/2} < 1$. The area shaded by lines oriented NE-SW at the right of each figure corresponds to the region excluded by BBN constraints on NLSP to gravitino decay. White color indicates $\Omega_{3/2} > 1$, i.e., overclosure of the Universe by

gravitinos. Finally the area marked with horizontal lines is unphysical as it corresponds to $F_S > F$ ($k > 1$).

We choose 0.01 and 1 as lower and upper bounds respectively to delimit where the gravitino can account for dark matter, even though cosmological data restrict this to a much smaller range. However the calculations presented here contain intrinsic uncertainties of factors of a few that were mentioned in the previous sections, and therefore the green, red and blue areas should be understood as indicative of the region in which one can find solutions for gravitino dark matter.

As indicated in the caption of Fig. 1, the left panels correspond to $M_X = 10^5 \text{ GeV}$ while the right panels correspond to $M_X = 10^{10} \text{ GeV}$ (for which the condition $k \leq 1$ translates in $m_{3/2} \gtrsim 250 \text{ keV}$). The upper and lower panels correspond respectively to a stau and a bino NLSP. As anticipated in the previous Section, the BBN constraints on hadronic and electromagnetic energy injection do not apply to the stau at these small values of $m_{3/2}$, as a result of the low stau relic abundance. In fact, for a stau of mass $M_{\text{NLSP}} \simeq 150 \text{ GeV}$, a gravitino as heavy as $m_{3/2} \sim 10 \text{ GeV}$ is allowed by BBN [17]. However, the constraints are quite stringent for the case of the bino NLSP, and result in an upper bound $m_{3/2} \lesssim 10 \text{ MeV}$ for $M_{\text{NLSP}} = 150 \text{ GeV}$. At a fixed value of the NLSP relic abundance, the BBN constraints give an upper bound on the decay timescale; hence, the above limit on $m_{3/2}$ scales as $M_{\text{NLSP}}^{5/2}$, see Eq. (15). The BBN limit on $m_{3/2}$ evolves as follows with respect to the bino annihilation cross-section; for reference, we recall that the cross-section used in the calculations reported in Fig. 1 is $\langle \sigma_{\text{NLSP} \nu} \rangle \simeq \sigma_0 (M_{\text{NLSP}}/100 \text{ GeV})^{-2}$ with $\sigma_0 = 10^{-9} \text{ GeV}^{-2}$ and $M_{\text{NLSP}} = 150 \text{ GeV}$. If σ_0 is decreased by a factor 10 to 100, the upper bound on $m_{3/2}$ shifts to $\sim 100 \text{ MeV}$; if, conversely, the cross-section is increased by a factor 10 to 100, the upper bound on $m_{3/2}$ shifts to $\sim 3 \text{ MeV}$. As mentioned in the previous section, we have implicitly assumed a branching ratio to hadronic decay $B_{\text{had}} = 0.15$; if the bino is nearly degenerate in mass with Z , the hadronic decay mode is suppressed, with a value possibly as small as $B_{\text{had}} \sim 10^{-3}$. In this case, for σ_0 chosen as above, the bound on $m_{3/2}$ would be $\sim 100 \text{ MeV}$, increasing to $\sim 10 \text{ GeV}$ if σ_0 is increased by a factor 100, and remaining constant if σ_0 is decreased by a factor as large as 100. Overall the BBN constraints result in a bound $m_{3/2} \lesssim 10 \text{ MeV} \rightarrow 1 \text{ GeV}$ depending on the bino mass, annihilation cross-section and hadronic branching ratio.

Figure 1 illustrates the so-called light gravitino problem: if $m_{3/2} \gtrsim 1 \text{ keV}$, gravitinos overclose the Universe and/or disrupt BBN unless the reheating temperature is low, $T_{\text{RH}} \sim \min [10^8 m_{3/2} (M_3/1 \text{ TeV})^{-2}, M_X/10]$, and gravitinos are not too heavy, $m_{3/2} \lesssim 10 \text{ MeV} - 1 \text{ GeV}$. Although it is not impossible to achieve such small reheating temperatures, either by low-scale inflation or a late phase of thermal inflation, it is not particularly attractive as it puts further non-trivial requirements on the model and may pose problems for a successful genesis of

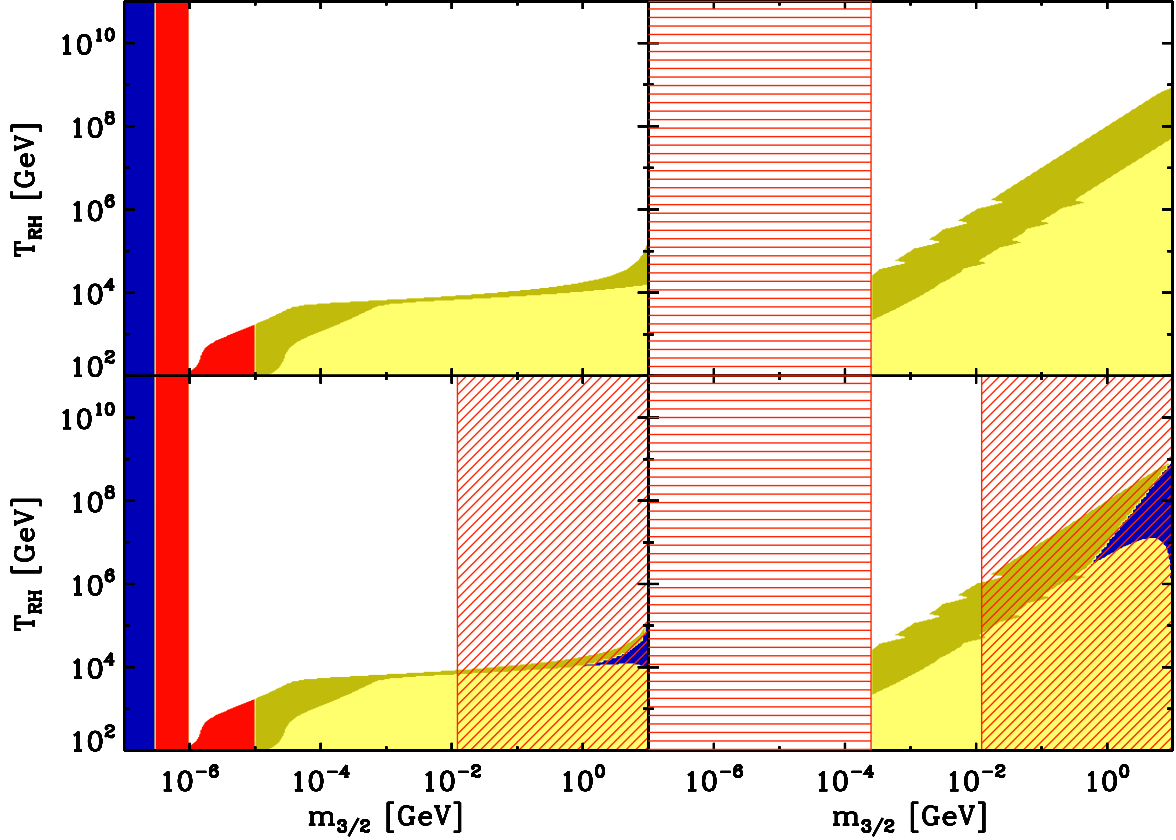


FIG. 1: Contours of $\Omega_{3/2}$ in the plane $T_{\text{RH}} - m_{3/2}$ for “fast” messenger decay, as discussed in the text. White corresponds to $\Omega_{3/2} > 1$; increasingly lighter shaded areas (blue, red and green respectively) from left to right (in the left panel) correspond to hot, warm or cold dark matter respectively with $0.01 \leq \Omega_{3/2} \leq 1$. The lightest zone (yellow) indicates $\Omega_{3/2} < 0.01$. The area shaded (in red) by lines oriented NE-SW is the zone excluded by BBN constraints on NLSP decay. The zone shaded with horizontal lines is theoretically excluded as it corresponds to $F_S > F$. The NLSP is bino-like in the lower panels, and stau-like in the upper panels. The messenger mass scale is $M_X = 10^5$ GeV in the left panels and $M_X = 10^{10}$ GeV in the right panels.

baryon or lepton asymmetry. Moreover it is necessary to tune the reheating temperature to the gravitino and/or messenger mass, even though these quantities derive from sectors of the theory that are *a priori* unrelated.

The region $m_{3/2} \lesssim 1$ keV is devoid of constraints on T_{RH} since the gravitino is so light that even at thermal equilibrium, it cannot overclose the Universe. However such light gravitinos make up dark matter that is too warm to reproduce existing data on the large scale structures. Heavier gravitinos $m_{3/2} \gtrsim 10$ MeV are excluded by big-bang nucleosynthesis constraints if the NLSP is a bino. Since NLSPs decay at time $\tau_{\text{NLSP}} \sim 6 \times 10^4 \text{ sec} (M_{\text{NLSP}}/100 \text{ GeV})^{-5} (m_{3/2}/1 \text{ GeV})^2$, the heavier the gravitino the later and the more constrained the decay. Figure 1 differs from those shown in Refs. [4, 6] because we have included constraints from GMSB phenomenology (namely $k \leq 1$) as well as updated constraints from BBN and structure formation, and a more accurate calculation of the gravitino relic abundance. Overall, one finds that the range of allowed $m_{3/2}$ is severely restricted when compared to previous studies.

In particular, the present conclusion is at variance with Ref. [6] which argued that for a sufficiently small messenger mass scale $M_X \sim 10^5$ GeV and sufficiently large gravitino mass $m_{3/2} \gtrsim 2$ GeV, it is possible to find solutions to the gravitino problem for arbitrarily high reheating temperatures. The discrepancy with [6] is tied to the neglect in that study of the SUGRA induced MSSM particles contribution to the gravitino abundance at large reheating temperatures, as discussed in Section III.A., as well as of the big-bang nucleosynthesis constraints on NLSP decay. As can be seen in the left panels of Fig. 1, the upper bound on T_{RH} does indeed shift upwards, albeit for larger $m_{3/2}$ than expected in [6], i.e. $m_{3/2} \gtrsim 10$ GeV for $M_X \sim 10^5$ GeV. This $m_{3/2}$ region is however forbidden by BBN constraints on energy injection, for both stau and bino NLSP. If the gluino mass scale is smaller than the fiducial value of 1 TeV, say $M_3 = 300$ GeV, this region of parameter space where the bound on T_{RH} is relaxed, moves to smaller $m_{3/2}$, i.e. $m_{3/2} \gtrsim 0.4 - 1$ GeV. However, the BBN constraints also move to smaller $m_{3/2}$ since the NLSP mass is reduced as the gluino mass. Finally, since

the MSSM particle contribution to $\Omega_{3/2}$ contains a dependence on $m_{3/2}$ and T_{RH} , we find that T_{RH} is always bounded from above due to the BBN bound on $m_{3/2}$; for instance, for $M_3 = 300 \text{ GeV}$ and for a stau NLSP, the maximal reheating temperature where $\Omega_{3/2} < 1$, is $\sim 10^9 \text{ GeV}$. In the end it turns out that some fine-tuning between M_3 , M_{NLSP} and $m_{3/2}$ is required to find a region in which T_{RH} can become as large as $\sim 10^9 \text{ GeV}$.

There exist other potential renormalizable interaction terms that violate messenger number by one unit. (Such operators lead to $1/m_{\text{P1}}^2$ suppressed proton decay [18], due to gauge coupling of the messenger fields.) In the following we assume for definiteness that messengers come in complete representations of $SU(5)$, in particular, as $\mathbf{5}_M + \bar{\mathbf{5}}_M$ (or $\mathbf{10}_M + \bar{\mathbf{10}}_M$), while the visible sector superfields are denoted by $\bar{\mathbf{5}}_F + \mathbf{10}_F$, and the Higgses sit in one pair of $\mathbf{5}_H + \bar{\mathbf{5}}_H$ and one $\mathbf{24}_H$ supermultiplets. Gauge symmetry limits those interaction terms between messengers and visible sectors superfields in the superpotential to the following:

$$W_{\text{ren}} \supset \left\{ \bar{\mathbf{5}}_M \bar{\mathbf{5}}_{F,H} \mathbf{10}_F, \mathbf{5}_M \mathbf{10}_F \mathbf{10}_F, \mathbf{5}_M \bar{\mathbf{5}}_{F,H} \mathbf{24}_H, \right. \\ \left. \bar{\mathbf{5}}_M \mathbf{5}_H \mathbf{24}_H, \bar{\mathbf{10}}_M \mathbf{5}_H \mathbf{5}_H, \mathbf{10}_M \bar{\mathbf{5}}_{H,F} \bar{\mathbf{5}}_{H,F}, \right. \\ \left. \mathbf{10}_M \mathbf{10}_F \mathbf{5}_H, \mathbf{10}_F \bar{\mathbf{10}}_M \mathbf{24}_H \right\}. \quad (23)$$

The interaction terms considered in Ref. [43] are contained in $\bar{\mathbf{5}}_M \bar{\mathbf{5}}_H \mathbf{10}_F$ and $\mathbf{10}_M \mathbf{10}_F \mathbf{5}_H$, and lead to fast messenger decay. Slow decay may, in principle, result from operators in Eq. (23) which involve particles with GUT scale masses. However, renormalizable couplings to a $\mathbf{24}_H$ must be excluded as they lead to M_{GUT} mass mixings between messengers and visible sectors particles, hence they would spoil the phenomenology at the electroweak scale. The colored triplet GUT Higgs in $\mathbf{5}_H$ and $\bar{\mathbf{5}}_H$ carry masses of order M_{GUT} (but no *vev*), but they do not couple to the lightest messenger, which is either the 5th component of a $\mathbf{5}_M$ or the (4, 5) component of a $\mathbf{10}_M$. Hence the colored Higgses do not lead to suppression of the decay width. In Ref. [7] it was proposed that delayed messenger decay could occur if the decay of the lightest messenger was suppressed by the mediation of a particle of mass $\sim 10^{12} \text{ GeV}$ in a renormalizable interaction. Our present discussion shows that this model is not natural in the sense that it requires a new particle with both the required mass $\sim 10^{12} \text{ GeV}$ and gauge charges such that the required renormalizable interaction could occur. In the above list of possible interactions terms, such coupling does not appear for the minimal content of $SU(5)$.

One may also argue that the required fine-tuning to avoid flavor changing neutral currents (in particular for light messengers) may actually indicate the absence of those renormalizable interactions, and that messenger number violation occurs via further suppressed interactions. Such couplings will be discussed further below.

2. Renormalizable couplings in the Kähler function

Fujii & Yanagida [8] have proposed messenger-matter mixing due to a correction in the superpotential $\delta W \simeq (\langle W \rangle / m_{\text{P1}}^2) \mathbf{5}_M \bar{\mathbf{5}}_F$, where $\langle W \rangle \simeq m_{3/2} m_{\text{P1}}^2$. In the framework of supergravity, a possible origin for such a superpotential term can be highlighted by adding to the minimal Kähler potential $K_0 = \sum_i \Phi_i^\dagger \Phi_i$, (i running over all superfields Φ_i) a non-minimal part δK given by,

$$\delta K = \mathbf{5}_M \bar{\mathbf{5}}_F + h.c. \quad (24)$$

δK is allowed by gauge symmetries (and possibly by an R-symmetry as well, for conveniently chosen R charges). Then, making use of the usual invariance of the supergravity Lagrangian under Kähler transformations $K \rightarrow K + F(\Phi) + F^*(\Phi^*)$, followed by superpotential $W \rightarrow e^{-F} W$ (and super-Weyl) scalings, the above δW is obtained for $F(\Phi) = -\delta K$ to the lowest order in $1/m_{\text{P1}}^2$, provided that a constant is added to the superpotential to fine-tune the cosmological constant after supersymmetry breaking (whence $\langle W \rangle \simeq m_{3/2} m_{\text{P1}}^2$)⁵. In our notations, the lightest messenger ϕ is a linear combination of the lightest scalar components of $\mathbf{5}_M$ and $\bar{\mathbf{5}}_M^*$, see Section 2.1. The mixing between $\mathbf{5}_M$ and $\bar{\mathbf{5}}_F$ generated from $\delta W \simeq m_{3/2} \mathbf{5}_M \bar{\mathbf{5}}_F$ thus leads to the decay of X into a lepton and a gaugino with width $\Gamma_{X \rightarrow l\lambda} \simeq (g^2/16\pi) m_{3/2}^2/M_X$ [8].

We should stress here that, starting as we do from δK rather than δW of [8], one expects further contributions to the messenger decay or annihilation, with possibly important effects on the final gravitino abundance. Indeed, other contributions to the decay into visible sector particles originate from the supergravity scalar potential [29]

$$V_B = e^{K/m_{\text{P1}}^2} \left[K^{ij*} \left(W \frac{K_i}{m_{\text{P1}}^2} + W_i \right) \left(W^* \frac{K_{j^*}}{m_{\text{P1}}^2} + W_{j^*}^* \right) - \frac{3WW^*}{m_{\text{P1}}^2} \right] \quad (25)$$

where, i, j^* label the full set of scalar fields $\phi_i, \phi_{j^*}^*$; K^{ij*} denotes the inverse of the matrix $\partial K / \partial \phi_i \partial \phi_{j^*}^*$, and $W_i = \partial W / \partial \phi_i$, $W_{j^*}^* = \partial W^* / \partial \phi_{j^*}^*$. From $K \supset K_0 + \delta K$ and taking for illustration the case of $\mathbf{5} + \bar{\mathbf{5}}$ messengers with $W \supset S \mathbf{5}_M \bar{\mathbf{5}}_M + y \bar{\mathbf{5}}_F \bar{\mathbf{5}}_H \mathbf{10}_F + \langle W \rangle$ one finds the leading contributions to the potential which induce the decay of the lightest messenger,

$$V_B \supset m_{3/2} S^* \bar{\mathbf{5}}_F \bar{\mathbf{5}}_M^* + y m_{3/2} \mathbf{5}_M \bar{\mathbf{5}}_H^* \mathbf{10}_F^* - 2m_{3/2}^2 \mathbf{5}_M \bar{\mathbf{5}}_F \\ + \frac{1}{m_{\text{P1}}^2} \left\{ \mathbf{5}_M \bar{\mathbf{5}}_F \left[|S|^2 (\mathbf{5}_M \mathbf{5}_M^* + \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M^*) \right. \right.$$

⁵ Altogether this is very reminiscent of the Giudice-Masiero mechanism which provides a solution to the so-called μ -problem [44].

$$\begin{aligned}
& + \mathbf{5}_M \bar{\mathbf{5}}_M|^2 + y^2 |W_{y|i}|^2 \Big] \\
& + (S^* \bar{\mathbf{5}}_M^* \bar{\mathbf{5}}_F + y \mathbf{5}_M \bar{\mathbf{5}}_H^* \mathbf{10}_F^*) (S \mathbf{5}_M \bar{\mathbf{5}}_M + y \bar{\mathbf{5}}_F \bar{\mathbf{5}}_H \mathbf{10}_F) \Big\} \\
& + h.c. \tag{26}
\end{aligned}$$

where we have neglected terms suppressed by higher powers of $1/m_{\text{P1}}^2$ or of order $m_{3/2}/m_{\text{P1}}^2$ and smaller. Other operators which do not induce messenger decay into standard model particles are not shown, being irrelevant for the present discussion. After the spurion scalar field S has developed a supersymmetric vev , the first term of Eq. (26) leads to the bilinear operator $m_{3/2} M_X \bar{\mathbf{5}}_F \bar{\mathbf{5}}_M^*$ contributing to the decay considered in [8] (actually a similar mixing between the fermionic partners is also generated, see below). Note that the second operator in Eq. (26) can mediate an equally efficient decay of the lightest messenger to a (colorless) Higgs and a slepton $\Gamma_{X \rightarrow H \tilde{f}} \simeq (y^2/32\pi) m_{3/2}^2/M_X$, provided that the Yukawa coupling is large (i.e. if δK involves the third generation). A similar decay to a Higgsino and a standard model fermion occurs as well, triggered by the MSSM Yukawa vertex and $m_{3/2} M_X \bar{\mathbf{5}}_F \bar{\mathbf{5}}_M^*$. In any case, these new contributions do not lead to a significant change in the analysis of the gravitino relic density, being of the same order as $\Gamma_{X \rightarrow \lambda \lambda}$.

Other contributions in Eq. (26) can potentially lead to important modifications when loop effects are considered. This is due on one hand to the supersymmetry preserving non-renormalizable operators of the form $\mathbf{5}_M \dots \phi \phi^* \dots \bar{\mathbf{5}}_F/m_{\text{P1}}^2$, with ϕ denoting an arbitrary field, and on the other hand to renormalizable operators which induce *hard* supersymmetry-breaking after cancellation of the cosmological constant. (In connection with the latter operators, the presence of the spurion, a visible sector singlet, could destabilize the hierarchy of the messenger and/or electroweak scales [45].) A thorough discussion of these issues which are relevant for the theoretical consistency of the effective supergravity model is out of the scope of the present paper. Hereafter we give only a partial and brief discussion of the two types of operators.

After the vev shift $S \rightarrow S + M_X$, the term of order m_{P1}^{-2} in Eq. (26) gives the operators

$$\left(\frac{M_X^2}{m_{\text{P1}}^2} \right) \times (\mathbf{5}_M \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M^* \bar{\mathbf{5}}_F, \mathbf{5}_M \bar{\mathbf{5}}_F \bar{\mathbf{5}}_M \bar{\mathbf{5}}_M^*, \mathbf{5}_M \bar{\mathbf{5}}_F \mathbf{5}_M \mathbf{5}_M^*).$$

These operators generate potentially very large corrections to the $\mathbf{5}_M - \bar{\mathbf{5}}_F$ mixing, through one-loop contributions of the $\mathbf{5}_M$ and $\bar{\mathbf{5}}_M$, which are of the order of $\Lambda^2 \sim m_{\text{P1}}^2$, where Λ is the cut-off scale of the physics underlying the effective supergravity Lagrangian. Even more, the operators containing SS^* instead of M_X^2 lead to more dangerous corrections of order Λ^4 due to two-loop diagrams. The same diagrams can also induce direct decays of the lightest messenger into MSSM particles, such as $X \rightarrow \gamma(Z) + \tilde{l}$. It is thus important to assess the supersymmetric cancellations which would

keep those contributions under control, eventhough they originate from the gravitational non-renormalizable sector. The companion operators involving the messenger fermions are contained in the $O(1/m_{\text{P1}}^2)$ part of

$$- \mathcal{L}_F \supset e^{K/2m_{\text{P1}}^2} \left[\frac{1}{2} \mathcal{D}_i D_j W \bar{\chi}_R^i \chi_L^j + h.c. \right] \tag{27}$$

where $\mathcal{D}_i D_j W = W_{ij} + (K_{ij}/m_{\text{P1}}^2)W + (K_i/m_{\text{P1}}^2)D_j W + (K_j/m_{\text{P1}}^2)D_i W - (K_i K_j/m_{\text{P1}}^4)W - \Gamma_{ij}^k D_k W$ (see [29]). They read

$$\begin{aligned}
- \mathcal{L}_F \supset \frac{1}{2m_{\text{P1}}^2} \Bigg[& \bar{\psi}_R \psi_L S (\mathbf{5}_M \bar{\mathbf{5}}_F + h.c.) \\
& + \bar{\psi}_R \mathbf{5}_M \psi_L \bar{\mathbf{5}}_F S \Bigg] + h.c. \tag{28}
\end{aligned}$$

where ψ denotes the Dirac field which combines the two fermionic components of the $\mathbf{5} + \bar{\mathbf{5}}$ messenger superfields. [The order of occurrence of the fields in Eq. (28) indicates how they are combined into $SU(5)$ invariants .] In the supersymmetric limit, the one-loop contributions to the $\mathbf{5}_M - \bar{\mathbf{5}}_F$ mixing induced by $-\mathcal{L}_F$ with $S \rightarrow \langle S \rangle$ cancel exactly the ones from the scalar loops discussed above. After SUSY breaking through the vev of the auxiliary field F_S , no dependence on the ultra-violet cut-off Λ is reintroduced. In particular, even $\log \Lambda$ terms cancel out (though they would not have altered the size of the mixing) yielding a correction of order F_S^2/m_{P1}^2 in the limit $F_S \ll M_X^2$, and of order M_X^4/m_{P1}^2 in the limit $F_S \simeq M_X^2$, which remains negligible when compared to the tree-level mixing magnitude $m_{3/2} M_X$, Eq. (26), in the parameter space region relevant for gravitino dark matter.⁶ Finally, as mentioned before, some *hard* SUSY breaking operators are generated in Eq. (27) from $m_{3/2} e^{K/2m_{\text{P1}}^2} (\frac{1}{2} \bar{\chi}_R^i \chi_L^j \delta K_{ij} + h.c.)$, leading to bilinear matter fermion mixing between the messengers and the MSSM particles, $m_{3/2} (\bar{\mathbf{f}}_R \psi_L + h.c.)$. These can potentially lead to quadratic divergences which would destabilise the scale of the mixing $\mathbf{5}_M - \bar{\mathbf{5}}_F$ between the lightest messenger and the MSSM scalar particles. However, leading one-loop (tadpole) effects with one mass insertion occur as corrections to $\bar{\mathbf{5}}_M^* - \bar{\mathbf{5}}_F$ and turn out to be at worse $O((\Lambda^2/m_{\text{P1}}^2) m_{3/2} M_X)$, thus harmless for a cut-off of order $\sim m_{\text{P1}}$. (Note also that the matter fermion kinetic terms induce fermionic tadpole contributions which cancel among themselves due to the derivative couplings.) To summarize, in the sector of the supergravity Lagrangian not involving the gravitino the δK piece of the Kähler potential leads essentially to the same

⁶ We checked also for cancellations to two-loop order considering subclasses of Feynman diagrams which involve the spurion scalar and fermion virtual contributions. A detailed study is outside the scope of the present paper.

features as discussed in [8], at least to leading order in $1/m_{\tilde{P}_1}^2$ and up to one-loop. One exception is the messenger decay induced by first term in Eq. (26) which we will consider later on.

The relic abundance for the gravitino can be calculated using the techniques developed in Section III and the results are shown in Fig. 2. This figure uses the color shading as in Fig. 1 but is plotted in the plane $m_{3/2} - M_X$ instead of T_{RH} , which was taken to be $T_{RH} = 10^{12}$ GeV. The plot shown in the left panel assumes that the spurion S is much heavier than the lightest messenger X , in which case annihilation $XX^* \rightarrow \tilde{G}\tilde{G}$ takes place with the cross-section given in Eq. (7). As discussed in Section II.B, this cross-section violates unitarity for $M_X \gtrsim 10^7 \text{ GeV} (m_{3/2}/1 \text{ keV})^{2/3}$, i.e. in the region above the dashed thick line in the left panel of Fig. 2. There is no solution in this case for gravitino dark matter, at variance with the conclusions of Ref. [8]. The main reason is that annihilation into goldstinos has not been accounted for in Ref. [8], yet the solution for gravitino dark matter proposed by these authors lies in the region in which unitarity is violated. The annihilation cross-section at its unitarity bound is much larger than that used in Ref. [8] for annihilation through gauge interactions, hence the messenger relic abundance and the amount of gravitino dilution are correspondingly smaller. At the very least, since one cannot predict the cross-section in this region where multi-goldstino production violates unitarity, one can conclude that the results for the scenario of Ref. [8] in this region are model dependent in that they require contributions from the hidden sector to bring down multi-goldstino production to a negligible level.

In the right panel of Fig. 2, it is assumed that S is much lighter than X . Annihilation into goldstinos scales with the effective Yukawa coupling F_S/F of the messenger to the goldstino component of the fermionic partner of S , as in Eq. (6). This annihilation channel thus contributes only in the region of direct GMSB scenarios where $k \equiv F_S/F \sim 1$, which happens to be that where gravitino dark matter can be found for the mixing term $\mathbf{5}_M \mathbf{\bar{5}}_F$ proposed in Ref. [8]. The lightest messenger can also annihilate into a pair of spurions, as discussed in Section II.B. Furthermore, one must also consider the possible decay of the lightest messenger into one visible sector sneutrino and two goldstinos, which is induced by the above mixing term and $XX^* \tilde{G}\tilde{G}$ four-point vertices similar to the ones discussed after Eq. (3). For instance, the latter vertex is induced by terms of the form $-e^{K/2m_{\tilde{P}_1}^2} W \tilde{\Psi}_L^\mu \sigma_{\mu\nu} \Psi_R^\nu / m_{\tilde{P}_1}^2$, [29], using Eqs. (24, 4) and $W \rightarrow \langle W \rangle$.

One expects this decay width to scale as $\Gamma_{X \rightarrow \tilde{\nu} \tilde{G}\tilde{G}} \sim 0.1 (F_S/F)^4 \Gamma_{X \rightarrow \lambda \lambda}$, with the prefactor of 0.1 accounting for the enlargement of phase space. This decay channel produces highly relativistic gravitinos which mix with the “cold” gravitinos produced by other channels, see Section III.A, resulting in mixed dark matter. The small area around $m_{3/2} \sim 10 - 100 \text{ keV}$ is shown in red (medium shading) in Fig. 2, indicating that the hot grav-

itinos contribute to more than 10% of the gravitino energy density and that the averaged velocity corresponds to warm dark matter. Finally one must also include decay into a spurion and a sneutrino induced by the three scalar coupling, first term in Eq. (26), with width $\Gamma_{X \rightarrow S \tilde{\nu}} \simeq (1/16\pi) m_{3/2}^2 / M_X$.

The inclusion of these various annihilation and decay channels modify the region of parameter space where one can find gravitino dark matter with respect to the conclusions of Ref. [8]. The various effects add up in pushing this region to higher values of M_X . In effect, the increased lightest messenger annihilation cross-section decreases their relic abundance hence the amount of entropy production; similarly, the increased lightest messenger decay width increases their decay temperature hence also decreases the entropy production. These effects can be compensated, at a given value of $m_{3/2}$, by increasing M_X since $\langle \sigma_{XX^* \rightarrow \dots \nu} \rangle \propto 1/M_X^2$ and $\Gamma_{X \rightarrow \dots} \propto 1/M_X$.

Note that Fig. 2 assumes that the NLSP is a bino; as seen previously, the BBN bounds would be relaxed if the NLSP turned out to be a stau.

Finally, the influence of the choice of the post-inflationary reheating temperature (taken as $T_{RH} = 10^{12}$ GeV here) is as follows. For this chosen value of T_{RH} , gravitinos (i.e. their 1/2 component) abundances are brought to equilibrium for all $m_{3/2} \lesssim 100 \text{ MeV}$. For smaller values of T_{RH} , the plot would thus look similar in the region where $M_X \lesssim T_{RH}$, unless T_{RH} is that low $T_{RH} \lesssim 10^{12} \text{ GeV} (m_{3/2}/0.1 \text{ GeV})^2$ that gravitinos are initially not in chemical equilibrium. For $M_X \gtrsim T_{RH}$, messengers are not produced at the post-inflationary reheating, there is no entropy production (gravitino dilution) and the gravitino abundance in the plane $m_{3/2} - T_{RH}$ then resembles that shown in the right panels of Fig. 1.

If the messengers sit in $\mathbf{10} + \mathbf{\bar{10}}$ representations of $SU(5)$, a similar mixing term $\mathbf{\bar{10}}_M \mathbf{10}_F$ can be induced, and leads to similar effects. Note that the mixing $\mathbf{5}_M \mathbf{\bar{5}}_F$ and $\mathbf{\bar{10}}_M \mathbf{10}_F$ are constrained by R -symmetry charge assignments. If a non-holomorphic mixing $\mathbf{\bar{5}}_M^* \mathbf{\bar{5}}_F$ (or $\mathbf{\bar{10}}_M^* \mathbf{\bar{10}}_F$) is allowed, the situation is quite different. In effect, the lightest messenger now mixes in the kinetic terms with the sneutrino and its decay is no longer suppressed by $m_{3/2}/M_X$. As a consequence, decay is much faster, $\Gamma \sim \mathcal{O}(M_X)$, and no entropy production occurs; the situation is then as shown in Fig. 1.

B. Non-renormalizable couplings

1. Superpotential interactions

It is possible that interactions of the type Eq. (22) are forbidden and that messengers may only decay via non-renormalizable operators in W , as discussed previously. Operators which may arise due to unknown Planck-scale physics, which respect the $SU(5)$ gauge symmetry and

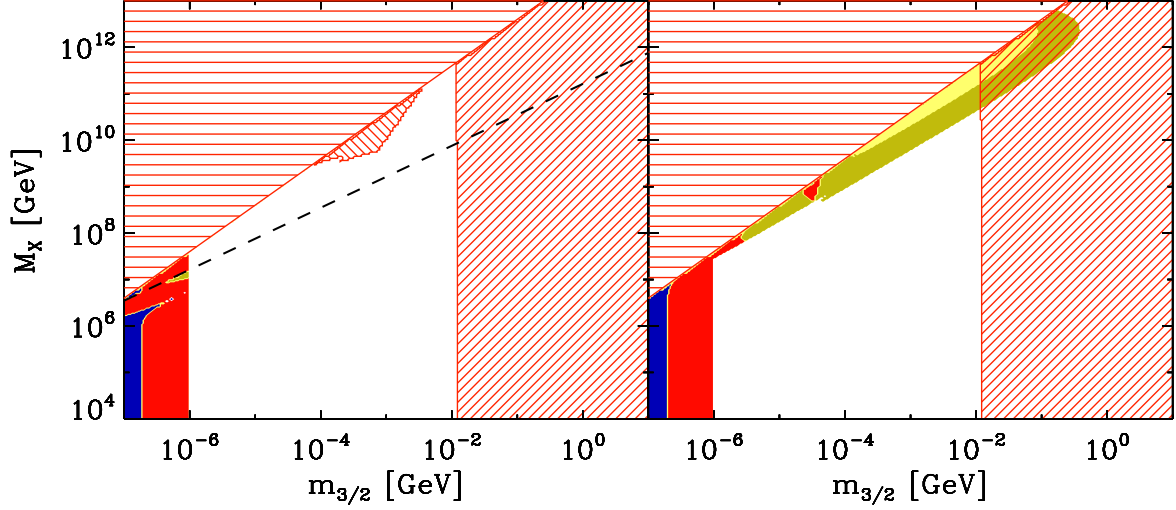


FIG. 2: Contours of $\Omega_{3/2}$ in the plane $M_X - m_{3/2}$ for one pair of messengers sitting in $\mathbf{5} + \bar{\mathbf{5}}$ representations; the lightest messenger X carries the gauge charges of a $\tilde{\nu}_L$, and decays into a slepton and a gaugino via the mixing term $\mathbf{5}_M \bar{\mathbf{5}}_F$ in the Kähler function. Color shading is as in Fig. 1; the NLSP is assumed to be a bino in both panels. In the left panel, the spurion S is assumed to be much heavier than the lightest messenger and annihilation $XX^* \rightarrow \tilde{G}\tilde{G}$ proceeds with the cross-section of Eq. 7; the dashed line indicates the limit above which this cross-section violates the unitarity bound, see Section II. In the right panel, the spurion S is assumed much lighter than X , and annihilation into goldstinos is given by Eq. 6. See text for further details.

that violate the messenger number by one unit are given to leading order in $1/m_{\text{Pl}}$, by:

$$W_{\text{non-ren}} \supset \frac{1}{m_{\text{Pl}}} \left\{ \begin{aligned} &\bar{\mathbf{5}}_M \mathbf{10}_F \mathbf{10}_F \mathbf{10}_F, \mathbf{5}_M \mathbf{5}_H \bar{\mathbf{5}}_{H,F} \bar{\mathbf{5}}_{H,F}, \\ &\bar{\mathbf{5}}_M \mathbf{5}_H \mathbf{5}_H \bar{\mathbf{5}}_{H,F}, \mathbf{5}_M \mathbf{5}_H \mathbf{5}_H \mathbf{10}_F, \\ &\bar{\mathbf{5}}_M \bar{\mathbf{5}}_H \mathbf{10}_F \mathbf{24}_H, \mathbf{5}_M \bar{\mathbf{5}}_{H,F} \mathbf{24}_H \mathbf{24}_H, \\ &\bar{\mathbf{5}}_M \mathbf{5}_H \mathbf{24}_H \mathbf{24}_H, \mathbf{10}_F \bar{\mathbf{10}}_M \mathbf{5}_H \bar{\mathbf{5}}_{H,F}, \\ &\bar{\mathbf{10}}_M \bar{\mathbf{5}}_{H,F} \bar{\mathbf{5}}_{H,F} \bar{\mathbf{5}}_{H,F}, \mathbf{10}_M \mathbf{5}_H \mathbf{5}_H \mathbf{5}_H, \\ &\mathbf{10}_M \bar{\mathbf{5}}_{H,F} \mathbf{10}_F \mathbf{10}_F, \mathbf{10}_M \mathbf{10}_F \mathbf{5}_H \mathbf{24}_H, \\ &\bar{\mathbf{10}}_M \mathbf{5}_H \mathbf{5}_H \mathbf{24}_H, \mathbf{10}_M \bar{\mathbf{5}}_{H,F} \bar{\mathbf{5}}_{H,F} \mathbf{24}_H, \\ &\bar{\mathbf{10}}_M \mathbf{10}_F \mathbf{24}_H \mathbf{24}_H, \mathbf{5}_M \bar{\mathbf{5}}_M \mathbf{5}_M \bar{\mathbf{5}}_F, \\ &\mathbf{10}_M \bar{\mathbf{10}}_M \mathbf{10}_M \mathbf{10}_F \end{aligned} \right\} \quad (29)$$

All terms in Eq. (29) which involve couplings of one lightest messenger to $\bar{\mathbf{5}}_F$ or $\mathbf{10}_F$ but not Higgses lead to decay into three-body final states with decay width $\sim 10^{-4} M_X^3 / m_{\text{Pl}}^2$. It is easy to see, using Eq. (19) that entropy production is not sufficient to dilute the gravitinos to the required abundance for a high post-inflationary reheating temperature $T_{\text{RH}} \gg 10^8 \text{ GeV}$ (and $T_{\text{RH}} \gg M_X$). Admittedly this is a drawback of the present scenario since those terms are the most generic.

Let us now consider the terms involving couplings to Higgses. For terms involving one $\mathbf{24}_H$ acquiring *vevs* of order $\sim M_{\text{GUT}}$, say $X \Phi_1 \Phi_2 H_{24} / m_{\text{Pl}}$, the Lagrangian contains the effective Yukawa interaction $\sim (M_{\text{GUT}} / m_{\text{Pl}}) X \bar{\chi}_1 \chi_2$ from the fermionic part of the Lagrangian contained in Eq. (27), with χ_1 and χ_2 the Weyl

spinors of Φ_1 and Φ_2 . This Yukawa interaction between the lightest messenger and two fermions with effective coupling constant $\sim 10^{-3}$ leads to fast decay if χ_1 and χ_2 have electroweak scale masses. Other terms in the Lagrangian lead to similar or only somewhat smaller partial widths. Consequently the conclusions of the previous section with regards to gravitino dark matter with "fast" decaying messengers apply. From Eq. (29) one can check that all possible above combinations involving one $\mathbf{24}_H$ contain a coupling of X to particles with electroweak masses except $\mathbf{10}_M \mathbf{10}_F \mathbf{5}_H \mathbf{24}_H$. However, even for the latter term, the scalar potential contains the interaction $\mathbf{10}_M \mathbf{10}_F \mathbf{24}_H y \mathbf{10}_F^* \mathbf{10}_F^* / m_{\text{Pl}}$ generated by $|\partial W / \partial \mathbf{5}_H|^2$, with y the third family Yukawa coupling. This interaction gives a decay width $\Gamma \sim 10^{-5} y^2 (M_{\text{GUT}} / m_{\text{Pl}})^2 M_X$, which is too large to allow solutions for gravitino dark matter.

Terms involving two $\mathbf{24}$ should be excluded as they lead to unacceptable mass mixings between messengers and visible sectors superfields.

Consider now terms of the form $X \Phi_1 \Phi_2 \Phi_3 / m_{\text{Pl}}$ containing at least one $\mathbf{5}_H$ but no $\mathbf{24}_H$. It can be checked that all terms of this form in Eq. (29) contain couplings of X to particles with electroweak masses, hence lead to fast decay as above, except for $\mathbf{5}_M \mathbf{5}_H \mathbf{5}_H \mathbf{10}_F / m_{\text{Pl}}$, $\mathbf{10}_M \mathbf{5}_H \mathbf{5}_H \mathbf{5}_H / m_{\text{Pl}}$ and $\bar{\mathbf{10}}_M \bar{\mathbf{5}}_{H,F} \bar{\mathbf{5}}_{H,F} \bar{\mathbf{5}}_{H,F} / m_{\text{Pl}}$. The first of these terms, when written for the lightest messenger, contains at least one colored Higgs, say H_i . However, the scalar potential term $|\partial W / \partial \mathbf{5}_{H_i}|^2$ generates here as well an interaction between the lightest messenger with 4 particles of electroweak masses, leading

to decay width $\Gamma \sim 10^{-5} y^2 M_X^3 / m_{\text{Pl}}^2$, still too large for dark matter solutions. The term $\mathbf{10}_M \mathbf{5}_H \mathbf{5}_H \mathbf{5}_H / m_{\text{Pl}}$ couples X to the three colored Higgses, hence its decay is too highly suppressed both by the GUT scale and phase space, $\Gamma \sim 10^{-12} (M_X / M_{\text{GUT}})^8 M_X^3 / m_{\text{Pl}}^2$, and cannot lead to decay before BBN for $M_X \lesssim 10^{14}$ GeV. Note that the big-bang nucleosynthesis constraints on NLSP decay can be translated into an extreme upper bound $M_X \lesssim 10^{12}$ GeV (see Fig. 2 and [40]). Finally, for the term $\mathbf{\overline{10}}_M \mathbf{\overline{5}}_{H,F} \mathbf{\overline{5}}_{H,F} \mathbf{\overline{5}}_{H,F} / m_{\text{Pl}}$, similar conclusions apply if three Higgses are involved. For couplings involving two colored Higgses, decay is also too highly suppressed. In effect, the lightest messenger can then decay into 5-body final state with mediation by a GUT mass Higgs, leading to $\Gamma \sim 10^{-6} y^2 (M_X / M_{\text{GUT}})^4 M_X^3 / m_{\text{Pl}}^2$. It cannot decay before BBN if $M_X \lesssim 10^{12}$ GeV, as can be checked using Eq. (16). Finally, if only one Higgs is involved in the coupling, the messenger can decay into four particle final states with decay width $\Gamma \sim 10^{-5} y^2 M_X^3 / m_{\text{Pl}}^2$ and yet provide no solution for gravitino dark matter.

The last two operators of Eq. (29) together with the first term of Eq (1) induce at one-loop order a mixing between the lightest messenger and the MSSM matter fields. This mixing is non-vanishing only after supersymmetry breaking and is found to be of magnitude $\sim (\sqrt{2} M_X / m_{\text{Pl}}) F_S \log(\Lambda / M_X)$. As discussed in Section IV.A.2, Λ is identified with a physical cut-off of order m_{Pl} , however, in contrast with the results of that section, there is here no (accidental) cancellation of the cut-off dependence and no suppression by the gravitino mass. The mixing is so large that decay is not accompanied by entropy production, and consequently there is no solution for gravitino dark matter.

In summary, non-renormalizable interaction terms in the superpotential for the minimal content of GMSB scenarios in $SU(5)$ grand unification do not allow for natural solutions leading to gravitino dark matter (unless the post-inflationary reheating temperature is tuned as before).

2. Non-renormalizable interactions in the Kähler function

Interaction terms in the Kähler potential K can either be holomorphic or not in the superfields, leading to different phenomenologies, which we explore in turn. To leading order in $1/m_{\text{Pl}}$, non-renormalizable holomorphic operators have the same form as those shown in Eq. (23),

$$K_{\text{hol}} = \frac{W_{\text{ren}}}{m_{\text{Pl}}} + h.c. \quad (30)$$

We write generically these operators as $X \phi_1 \phi_2 / m_{\text{Pl}}$. Let us assume for the moment that both ϕ_1 and ϕ_2 have masses of order of the electroweak scale.

The Kähler $U(1)$ connection $K_j \partial_\mu \phi_j - h.c.$, with $K_j \equiv \partial K / \partial \phi_j$ induces the following coupling to the fermionic components ψ^i of all the chiral superfields of the model

[29],

$$- \mathcal{L}_F \supset \frac{1}{2m_{\text{Pl}}^3} \bar{\psi}_L^i \gamma^\mu \psi_L^i \text{Im} \partial_\mu (X \phi_1 \phi_2), \quad (31)$$

which leads to a partial decay width $\propto M_X^7 / m_{\text{Pl}}^6$. From Eq. (27) an effective Yukawa coupling is generated,

$$\frac{\partial^2 K}{\partial \phi_1 \partial \phi_2} \frac{\langle W \rangle}{m_{\text{Pl}}^2} \bar{\psi}_R^1 \psi_L^2 \sim \frac{m_{3/2}}{m_{\text{Pl}}} X \bar{\psi}_R^1 \psi_L^2, \quad (32)$$

which leads to a highly suppressed partial decay width $\sim (m_{3/2} / m_{\text{Pl}})^2 M_X^3 / m_{\text{Pl}}^2$. Finally, couplings to goldstinos are generated notably by the gravitino mass term and gravitino kinetic terms:

$$\frac{-1}{2m_{\text{Pl}}} X \phi_1 \phi_2 \frac{W}{m_{\text{Pl}}^2} \Psi_\mu \sigma^{\mu\nu} \Psi_\nu + \epsilon^{\mu\nu\rho\sigma} \frac{1}{m_{\text{Pl}}^3} \partial_\rho (X \phi_1 \phi_2) \bar{\Psi}_\mu \bar{\sigma}_\nu \Psi_\sigma. \quad (33)$$

with the replacement $\Psi_\mu \rightarrow i\sqrt{2/3} \partial_\mu G / m_{3/2}$.

All terms lead to extremely slow decay if ϕ_1 and ϕ_2 have electroweak scale masses, and must be forbidden in order for the lightest messenger not to decay after BBN. However if one $\mathbf{24}_H$ is present, the replacement of this field by its *vev* in the Kähler function shows that one recovers a mixing term as proposed in Ref. [8] albeit with effective coupling $M_{\text{GUT}} / m_{\text{Pl}}$. This mixing term then leads to the same decay widths into one sfermion and one gaugino, or one sfermion and two goldstinos, as discussed before, albeit decreased by $(M_{\text{GUT}} / m_{\text{Pl}})^2 \sim 10^{-5}$. The consequences for gravitino dark matter are shown in Fig. 3. As discussed before, we should consider the cases where the spurion is heavier or lighter than the lightest messenger. The left panel shows the case where the spurion is heavier than the lightest messenger and the annihilation cross-section into goldstinos increases with increasing M_X to saturate at the unitarity bound above the dashed line. Only a small portion of parameter space allows for gravitino dark matter in this case; it is actually an amputated part of the solution shown in the right panel, see below. Above the dashed line, the results are highly uncertain since multi-goldstino is not well controlled. Moreover, the lightest messenger can decay into a sneutrino and a pair of goldstinos due to the above mixing, with a decay width which is expected to scale as $\Gamma_{X \rightarrow \tilde{\nu} \tilde{G} \tilde{G}} \propto (F_S^2 M_X^4 / F^4) \Gamma_{X \rightarrow l \lambda}$. The prefactor $(F_S^2 M_X^4 / F^4)$ denotes the effective coupling of XX^* to a pair of goldstinos in the heavy spurion limit. In the region in which unitarity is violated, it has been assumed that this effective coupling saturates at the value reached $[\sim \mathcal{O}(1)]$ when the annihilation cross-section reaches the unitarity bound. It is then comparable to the decay width into a lepton and a gaugino and produces highly relativistic gravitinos. The energy density contained in these gravitinos exceeds the BBN bounds on additional relativistic degrees of freedom so that most of this region is excluded, as indicated by the NW-SE oriented dashed lines in Fig. 3. The SW-NE oriented dashed lines exclude the part of parameter space at small $m_{3/2}$ in which the

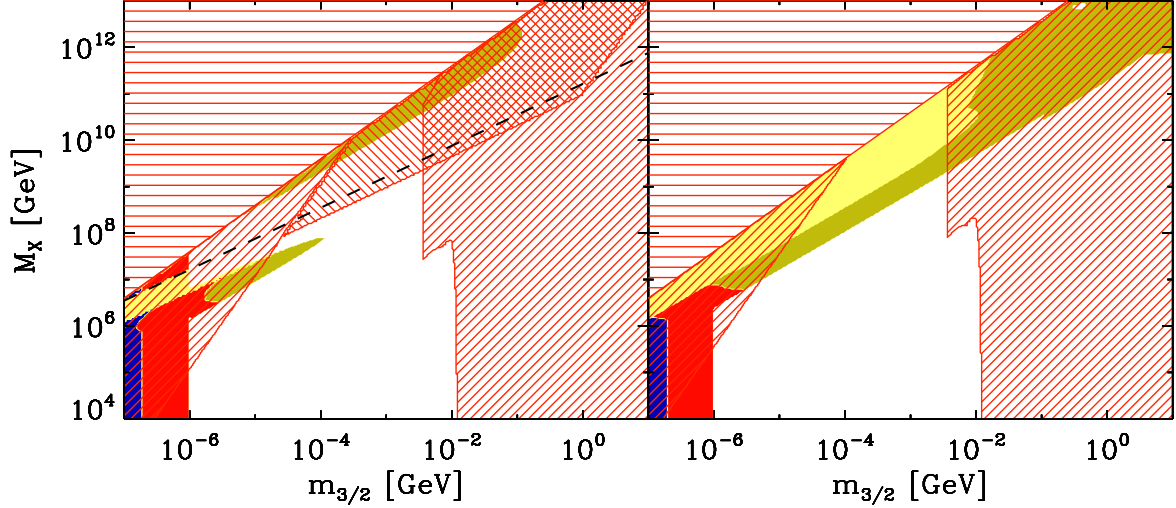


FIG. 3: Contours of $\Omega_{3/2}$ in the plane $M_X - m_{3/2}$ for one pair of messengers sitting in $\mathbf{5} + \bar{\mathbf{5}}$ representations; the lightest messenger X decays into two goldstinos and one sfermion or one sfermion and one gaugino via the mixing term $\sim X\phi_1(M_{\text{GUT}}/m_{\text{Pl}})$ in the Kähler function. Same color code and conventions as in Figs. 1, 2. The red NE-SW dashed area in the left part of the figure results from big-bang nucleosynthesis constraints on lightest messenger decay and the NW-SE area is forbidden by the contribution of gravitinos to the energy density at BBN; see text for further details.

lightest messenger decay occurs so late that it is forbidden by BBN constraints on energy injection. If the NLSP were a stau, this region would still be forbidden but the constraints at large $m_{3/2}$ would be relaxed.

In the right panel of Fig. 3, the spurion is assumed to be lighter than the lightest messenger and annihilation into goldstinos is less effective. One finds a solution for gravitino dark matter in a large part of parameter space, $m_{3/2} \sim 10 \text{ keV} \rightarrow 1 \text{ MeV}$, for scenarios of indirect gauge mediation, i.e. $k \ll 1$. This solution is the same as that discussed in Section IV.A.2, albeit shifted to smaller values of M_X ; this can be understood from the fact that for a same relic abundance of X , entropy production is larger in the present case since the decay width is further suppressed. Hence one can tolerate a smaller relic abundance, or equivalently a higher annihilation cross-section, i.e. a smaller M_X .

Finally consider now non-holomorphic non-renormalizable couplings between X and visible sector particles in K . Such couplings can take the form:

$$K \supset \frac{1}{m_{\text{Pl}}} \{ \mathbf{5}_M^\dagger \bar{\mathbf{5}}_{H,F} \mathbf{10}_F, \bar{\mathbf{5}}_M \mathbf{5}_H^\dagger \mathbf{10}_F, \bar{\mathbf{5}}_M^\dagger \mathbf{10}_F \mathbf{10}_F, \\ \mathbf{5}_M^\dagger \mathbf{5}_H \mathbf{24}_H, \mathbf{5}_M \mathbf{5}_H^\dagger \mathbf{24}_H, \bar{\mathbf{5}}_M \bar{\mathbf{5}}_{H,F}^\dagger \mathbf{24}_H, \\ \bar{\mathbf{5}}_M^\dagger \bar{\mathbf{5}}_{H,F} \mathbf{24}_H, \mathbf{10}_M^\dagger \mathbf{5}_H \mathbf{5}_H, \mathbf{10}_M \bar{\mathbf{5}}_{H,F}^\dagger \mathbf{5}_H, \\ \mathbf{10}_M^\dagger \bar{\mathbf{5}}_{H,F} \bar{\mathbf{5}}_{H,F}, \mathbf{10}_M^\dagger \mathbf{10}_F \mathbf{5}_H, \mathbf{10}_M \mathbf{10}_F \bar{\mathbf{5}}_{H,F}^\dagger, \\ \mathbf{10}_M^\dagger \mathbf{10}_F \mathbf{24}_H, \mathbf{10}_M \mathbf{10}_F^\dagger \mathbf{24}_H, + \text{h.c.} \} \quad (34)$$

As before, we write this coupling as $X^* \phi_1 \phi_2 / m_{\text{Pl}}$ and assume for the moment that ϕ_1 and ϕ_2 carry electroweak scale masses. Then decay into ϕ_1 and ϕ_2 with width

$\Gamma \sim 10^{-2} M_X^3 / m_{\text{Pl}}^2$ occurs via the mixing of kinetic terms between ϕ_1 and X or between ϕ_2 and X . As seen before, such a decay width does not lead to solutions for gravitino dark matter as entropy production is not significant. Inspection of Eq. (34) reveals that all terms fall in the above category except those involving one $\mathbf{24}_H$ as well as $\mathbf{10}_M^\dagger \mathbf{10}_F \mathbf{5}_H$ and $\mathbf{10}_M \mathbf{10}_F \bar{\mathbf{5}}_H^\dagger$.

The latter two terms necessarily contain one colored Higgs with GUT mass but no v_{ev} , which we assume to be ϕ_2 . Then the scalar potential term involving the inverse of the Kähler metric g contains a coupling of X to visible sector particles: $g^{\phi_2 M^*} D_{\phi_2} W D_{M^*} W^* \supset (\phi_1 / m_{\text{Pl}}) y M_X \mathbf{10}_F \mathbf{10}_F X$. This leads to decay into three-body final state with width $\Gamma \sim 10^{-4} y^2 M_X^3 / m_{\text{Pl}}^2$, again too large to yield solutions for gravitino dark matter.

Finally, if coupling to one $\mathbf{24}_H$ occurs, say ϕ_2 , mass mixing of order $M_{\text{GUT}} M_X$ occurs between ϕ_1 and X and leads to one negative mass squared eigenstate; this coupling must therefore be excluded.

V. DISCUSSION

In this section we explore qualitatively other possible avenues which may help reconcile gravitino dark matter with more generic GMSB scenarios. Indeed, the previous discussion has shown that interesting solutions for gravitino dark matter and/or the gravitino problem in GMSB with $SU(5)$ grand unification with a high post-inflationary reheating temperature, can only be found for some very specific couplings between the messenger and visible sector and in some restricted regions of param-

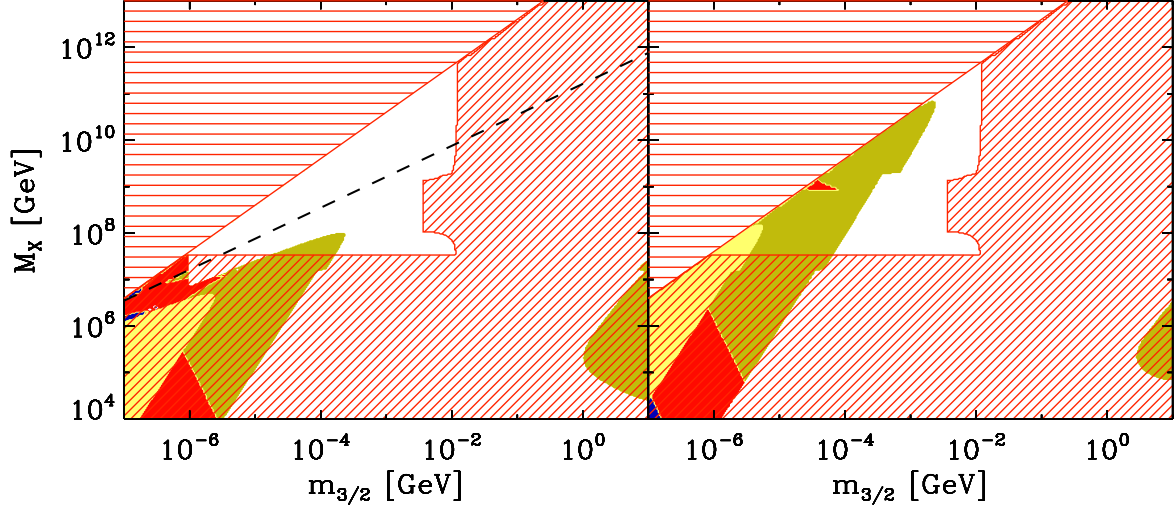


FIG. 4: Contours of $\Omega_{3/2}$ in the plane $M_X - m_{3/2}$ for one pair of messengers sitting in $\mathbf{5} + \bar{\mathbf{5}}$ representations as in Fig. 2, but for decay width $\Gamma \sim 10^{-10} M_X^3/m_{\text{P1}}^2$. See text for details.

ter space. It is furthermore necessary to assume that the spurion is much lighter than the lightest messenger so that multi-goldstino production remains at a safe level. A gravitational decay width $\Gamma \sim 10^{-3} M_X^3/m_{\text{P1}}^2$, which is generic in the sense that it is generated by most allowed non-renormalizable messenger-matter interaction terms, does not lead to satisfying solutions for gravitino dark matter.

It is instructive to consider the case of decay widths with the same scaling but whose prefactor is much smaller, $\Gamma \sim \epsilon M_X^3/m_{\text{P1}}^2$ with $\epsilon \ll 10^{-3}$. Figure 4 shows the solution for $\epsilon = 10^{-10}$. Such decay width can be achieved by terms of the form $W \supset X \Phi_1 \Phi_2 \Phi_3 H_{24}/m_{\text{P1}}^2$, with H_{24} designing a Higgs with non-zero v_{ev} in $\mathbf{24}_H$, or by most non-renormalizable couplings to order $1/m_{\text{P1}}$ discussed in the previous section, provided they are further suppressed by a factor $\sim 10^{-7}$. In this figure, one finds that in the left panel, where S is assumed heavier than X , i.e. where multi-goldstino production plays a significant role, there is room for gravitino dark matter only in a very limited region of parameter space. In this area, furthermore, the post-decay reheating temperature is quite close to 1 MeV. On the contrary, in the right panel one recovers a solution for gravitino dark matter with mass $m_{3/2} \sim 10 \text{ keV} \rightarrow 10 \text{ MeV}$ in direct gauge mediated scenarios $k \lesssim 1$. Higher values of the coupling ϵ lead to solutions shifted to higher M_X , with the dashed region due to late messenger decay shifting downwards. Lower values of ϵ lead to solutions shifted to smaller values of M_X , but with the excluded dashed region moving upwards in M_X .

Dangerous operators involving GUT-scale v_{ev} 's may also be present. However, global continuous R-symmetries are expected to play an important rôle in scenarios of supersymmetry breaking [46], and in a generic

setting they could control the absence of unwanted operators for properly chosen R-charges [47]. Discrete Z-symmetries, motivated by the need to improve the fine-tuning issues [48], can also play a selective rôle. In particular, the spurion gauge singlet superfield S can be present or not in non-renormalizable operators in the Kähler or superpotential depending on its R - and Z -charges attributions. Such operators, provided that they do not destabilize the mass hierarchies [49], [45], would lead to tree-level suppressions of the form $(\langle S \rangle/m_{\text{P1}})^{2n} \sim (M_X/m_{\text{P1}})^{2n}$. However the decay width is now too suppressed to yield reheating before BBN except in a very narrow region centered on $m_{3/2} \sim 1 \text{ MeV}$ and $M_X \sim 10^{10} \text{ GeV}$.

In contrast, a larger number of seemingly generic solutions to the gravitino problem/gravitino dark matter may be found if one considers $SO(10)$ grand unification and $M_S > M_X$. This case is studied analytically in a companion paper [26] for $M_X \sim 10^6 \text{ GeV}$. In Section II, it has been shown that the amount of entropy produced depends directly on the relic abundance of the lightest messenger, which in turn depends directly on its nature. In $SO(10)$, the lightest messenger is a $SU(3) \times SU(2) \times U(1)$ singlet ($\tilde{\nu}_R$ -like), hence its annihilation cross-section is suppressed: it may either annihilate through one-loop diagrams or at tree level into goldstinos, and (at tree level) through suppressed diagrams of GUT mass bosons exchange. One may estimate the relic abundance of the lightest messenger in this case by using the dimensional estimate $\langle \sigma_{XX \rightarrow \dots v} \rangle \sim (\alpha/4\pi)^4/M_X^2$ for one-loop diagrams and the annihilation cross-section into goldstinos computed in Section II.B (see also Ref. [26]). One finds that the relic abundance is larger than for the $SU(5)$ case, hence the amount of entropy production is expected to be correspondingly larger. One then finds that a generic

gravitational decay width $\Gamma \sim 10^{-3} M_X^3/m_{\tilde{P}_1}^2$ can lead to natural solutions for gravitino dark matter in a significant part of parameter space, as shown in Fig. 5.

This solution is attractive for several reasons. As seen in the left panel of Figs. 5, gravitino cold dark matter with the right relic abundance occurs in a rather large part of parameter space, hence it appears “natural” in this sense. Moreover the decay width to sparticles assumed $\Gamma \sim 10^{-3} M_X^3/m_{\tilde{P}_1}^2$ is generic as it is predicted by most non-renormalizable operators that violate messenger number by one unit. In this region of parameter space, the factor $k \equiv F_S/F \approx 10^{-5} - 10^{-2}$, hence gravitino dark matter would be obtained for indirect gauge mediated scenarios; in Ref. [26] it is argued that this case can be naturally incorporated in the simplest indirect GMSB scenarios [2]. Furthermore, the solution for gravitino dark matter occurs in the predictive region in which multi-goldstino production satisfies the unitarity bound, unlike the solutions seen hitherto for $SU(5)$. Finally, Fig. 5 assumes, as the previous figures, that the NLSP is a bino; for a stau NLSP, the BBN constraints at large $m_{3/2}$ would be relaxed, and this would enlarge in turn the space of solutions for $\Omega_{3/2}$.

One can show [26] that the amount of entropy production does not hinder successful leptogenesis at high reheating temperatures; this is all the more interesting as leptogenesis scenarios typically operate in models of $SO(10)$ grand unification rather than $SU(5)$. Strictly speaking, the above scenario requires the spurion field to be heavier than the lightest messenger. This issue is however model-dependent as was briefly discussed in section II.B. For completeness, we illustrate in the left panel of Fig. 5 the opposite configuration where the lightest messenger annihilation into a pair of spurion fields is controlled by Eq.(8). This annihilation leads to a too low messenger relic density for the entropy dilution mechanism to work.

Finally we note that the results obtained in the present study remain valid when R -parity is violated. In effect, in this case the gravitino lifetime is $\tau_{3/2} \sim 10^{20} \text{ sec } (m_{3/2}/1 \text{ GeV})^{-3}$ for trilinear R -parity violating terms [15] or $\tau_{3/2} \sim 10^{27} \text{ sec } (m_{3/2}/1 \text{ GeV})^{-3}$ for bilinear R -parity violating terms [14]. Hence the gravitino is sufficiently long-lived that it can be considered as stable dark matter with respect to the formation of large-scale structure. If the gravitino lifetime $\gtrsim 10^{27} \text{ sec}$ one also finds that distortions of the diffuse backgrounds due to gravitino decay are evading observational constraints. For trilinear R -parity violating terms, this requires $m_{3/2} \lesssim 10 \text{ MeV}$, while for bilinear terms, $m_{3/2} \lesssim 1 \text{ GeV}$ is sufficient. With regards to the NLSP, its decay can proceed into visible sector particles on a short timescale and BBN constraints can be evaded, albeit they are replaced with constraints on diffuse background distortions. Hence the plots in parameter space would look similar to what has been found above.

VI. CONCLUSIONS

We have presented an exploratory though detailed investigation of relic LSP gravitino abundances in scenar-

ios of gauge mediated supersymmetry breaking (GMSB). This study focuses on the possibility of gravitino dark matter and on solving the light gravitino overproduction problem for reheating temperatures after inflation that are “arbitrarily” high. GMSB scenarios contain intermediate mass scale $10^4 \text{ GeV} \lesssim M_X \lesssim 10^{12} \text{ GeV}$ messenger fields which by virtue of their gauge interactions are easily produced in the primordial plasma. Cosmology requires these particles to subsequently decay as they would otherwise overclose the Universe (except for a lightest messenger with $M_X \sim 10 - 30 \text{ TeV}$). Flavor-changing neutral currents impose somewhat restrictive limits on messenger number violating Yukawa interactions, possibly arguing for such messenger number violation to be rather weak. If so, the delayed decay of messengers may subsequently dilute any pre-existing gravitino abundances in accord with cosmological constraints.

We have thus investigated a fairly complete set of renormalizable and non-renormalizable messenger number violating operators within supersymmetric unification in $SU(5)$ (as well as some within $SO(10)$) and their impact on relic gravitino abundances. Results are shown for a variety of operators and imposing relevant constraints on NLSP decay and messenger decay from BBN, as well as constraints on the “warmness” of gravitino dark matter from the required successful formation of large-scale structure. With respect to prior, less detailed, studies [7, 8, 9, 10, 14, 18], we have uncovered a number of significant changes, notably the importance of messenger-messenger annihilation into two goldstinos in part of the $M_X - m_{3/2}$ parameter space, which modifies the messenger pre-decay freeze-out abundances in $SU(5)$ and $SO(10)$ grand unification.

In general, we have found that gravitino dark matter in $SU(5)$ grand unification in scenarios with high post-inflationary reheating temperatures T_{RH} is only possible for a few specific messenger-matter couplings. Furthermore we have shown that these models predict gravitino dark matter in regions of parameter space in which messengers annihilation to goldstinos violates unitarity unless one makes specific assumptions on the mass spectrum of GMSB models, and in particular, that the spurion S be much lighter than the lightest messenger.

In contrast, in $SO(10)$ grand unification gravitino dark matter may be obtained for a variety of generic operators and in the predictive region of parameter space where multi-goldstino production is under control, as long as renormalizable messenger number violating interactions in the superpotential are absent [26]. We thus believe that gravitino dark matter in GMSB scenarios is a viable alternative to neutralino (and gravitino) dark matter in supergravity scenarios, and as such deserves further detailed study.

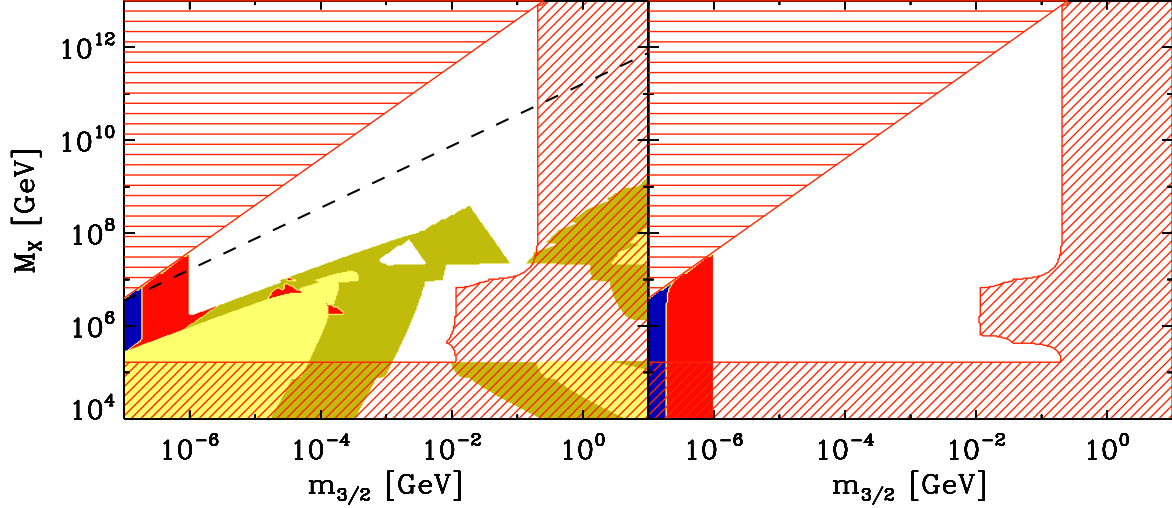


FIG. 5: Contours of $\Omega_{3/2}$ in the plane $M_X - m_{3/2}$ for one pair of messengers sitting in $\mathbf{16} + \overline{\mathbf{16}}$ representations of $SO(10)$; the lightest messenger X is a singlet under $SU(3) \times SU(2) \times U(1)$. Its loop-suppressed annihilation cross-section scales as $(\alpha/4\pi)^4/M_X^2$, and it decays into sparticles through non-renormalizable operators with width $\Gamma \sim 10^{-3} M_X^3/m_{\text{Pl}}^2$. Color shading is as in Figs. 1, 2. See text for details.

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